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# The Effect of Model Formulation on the Comparative Performance of Artificial Neural Networks and Regression

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# THE EFFECT OF MODEL FORMULATION ON THE COMPARATIVE PERFORMANCE OF ARTIFICIAL NEURAL NETWORKS AND REGRESSION

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# ABSTRACT

## THE EFFECT OF MODEL FORMULATION ON THE COMPARATIVE PERFORMANCE OF ARTIFICIAL NEURAL NETWORKS AND REGRESSION

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Multiple linear regression techniques have been traditionally used to construct predictive statistical models, relating one or more independent variables (inputs) to a dependent variable (output). Artificial neural networks can also be constructed and trained to learn these complex relationships, and have been shown to perform at least as well as linear regression on the same data sets. Research on the use of neural network models as alternatives to multivariate linear regression has focused predominantly on the effects of sample size, noise, and input vector size on the comparative performance of these two modeling techniques. However, research has also shown that a mis-specified regression model or an incorrect neural network architecture also contributes significantly to poor model performance. This dissertation compares the effects on model performance of various formulations of regression and neural network models, measuring performance in terms of mean squared error and variance. A factorial experiment is conducted in which model parameters are varied. Simulated data from three different functions are used to generate training and testing data sets. Statistical tests are used to determine differences in performance as well as the degree of model robustness, or the degree to which model performance is insensitive to changes in model formulation.

Based on the experimental results and conclusions, a predictive modeling methodology is proposed that capitalizes on the advantages of both neural network and regression approaches and assists practitioners in constructing accurate and robust predictive models.

This dissertation is dedicated to Almighty God
for His grace and strength, and for His abiding
love. With God, all things are possible.

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# CHAPTER I: INTRODUCTION

## Background

The heart of predictive modeling is the search for relationships between and among data. If a strong relationship is suspected to exist between two sets of data, a predictive mathematical model can be constructed that may be able to relate these two data sets in such a way that one can infer the properties of this relationship to new data, unrelated to the original set.

Multiple linear regression (MLR), a statistical data analysis technique, has been traditionally used to discover these data relationships by hypothesizing a type of functional relationship between these data (typically one or more independent variables and one dependent variable) and computing coefficients for the resulting equation. Researchers experimenting with neural computing and artificial neural networks (ANN) learned early on that these "black box" parallel computing architectures could solve regression problems without the requirement for a hypothesized regression function. By presenting the ANN with a sequence of input and desired output data examples, it learns the data relationship and can reproduce it with new data from the same population. A small, but growing body of research is attempting to understand how ANN can be used as a surrogate or an alternative to traditional predictive statistical model building techniques.

Multiple Linear Regression is one of the most popular and useful statistical tools available for quantitative analysis (Marquez, et al.. 1991). Through the process of minimizing the squared distance from the data points to the population mean, commonly called least squares estimation, MLR allows an analyst to build a parametric model, or curve, fitted to a set of data points. Such a curve is represented by a function relating one

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Journal Model: APA

or more independent variables to a dependent variable of interest. Armed with such a function, the analyst can, within the scope of the population being studied, generalize a predictive relationship between values of the independent variables and the dependent variable.

However, MLR has several limitations. Three important assumptions must be made concerning the distribution of the regression errors: they must be independent, normally distributed, and have a constant variance. But perhaps the most significant limitation of MLR is the requirement for an *a priori* hypothesis about the form of the function for which MLR will estimate the coefficients. The "true" functional relationship between the independent variables and the dependent variable is, of course, unknown. The analyst must study the data and provide a best estimate of this functional relationship. An analysis of the residual errors of the regression will show how well the hypothesized model explained the relationship of the data to the dependent variable. If the relationship is assumed to be linear, for example, and the true functional relationship is exponential, this mis-specification is reflected in a low value for the coefficient of determination, or R-squared, which is an indicator of how well the hypothesized model explains the relationship between the data.

Because the true, underlying functional relationship between the independent variables (inputs) and the dependent variable (outputs) is unknown, the analyst is never sure how much of the unexplained relationship is due to an under- or over-specified model, or simply variability in the data itself. A good predictive model should come as close as possible to discovering the theoretical function relating the input to the output variables.

Artificial Neural Networks (ANN) may be the tools that come closest to finding this relationship and improving the accuracy of predictive models. A typical ANN consists of a layer of one or more input nodes, called neurodes, a layer of one or more output neurodes, and may contain one or more hidden layers. Each of the neurodes in a layer is connected to every node in the adjacent layer, forming a "fully connected" network. Many types of ANN exist, including self-organizing maps, attractor networks and radial-basis function networks. However, the ANN being studied in this research are multilayer perceptrons. The term ANN, as used in this document, will refer to this type of network.

Neural networks differ from multiple regression in that the network learns the relationship between input and output responses through a process of changing weight values on the connections between the neurodes. Neural networks must be trained in order for them to learn these relationships between input and output patterns. For networks in which each input stimulus is related to a specific desired output, a series of example patterns is presented to the network along with the desired output. The output responses to the patterns are compared to the desired response and the resulting error is used to modify the weights on the interconnections between the neurodes. The patterns are repeatedly presented to the network until the error is minimized.

## Problem Statement

In recent years, practitioners and researchers in a number of fields have successfully used ANN as a surrogate for MLR in building predictive models, generally experiencing greater accuracy. However, while the use of ANN as an alternative to traditional statistical analysis methods appears promising, very little experimental research has been done to determine the conditions under which one technique may be more appropriate than the other. Controlled studies in which MLR and ANN models have been compared directly have concluded that there are situations in which regression models may be more appropriate. These studies examined the effects of data sample size and variability on the relative performance of regression models and ANN. There is general agreement that larger training samples (more data) produce better results, although there is some disagreement as to comparative performance when sample sizes are small. Some studies suggest that neural networks are unable to discover underlying relationships from data samples of fewer than 50 exemplars, while some have shown that ANN can discern patterns in training samples as small as 10 exemplars (Robinson, 1991; Marquez, et al., 1991; Markham and Rakes, 1998). Robinson (1991) concluded that training sample sizes greater then 50 are needed, although his conclusions are not supported by rigorous designed experiment.

There is also some disagreement over the significance of the size of the input vector on relative performance. Some studies conclude that neural networks should be able to handle a large number of cost drivers (independent variables) when used in cost estimating problems, and some imply that, as the size of the input vector increases, ANN should be a more attractive alternative to MLR (de-la-Garza and Rouhana, 1995; Smith and Mason, 1997). Another study disagrees, suggesting that a larger input vector creates an unnecessarily large network that could inhibit training speed and accuracy (Bode, 1998). It should be noted, however, that Bode's (1998) concern regarding longer computing times for large networks is largely a function of computing power. Expected future advancements in computing technology will likely make this issue less significant.

Although there are some conflicting conclusions regarding sample or input vector size, the effects of model formulation may overshadow the importance of these factors. Model formulation may play an even more significant role in the performance of regression and ANN models than training sample size, variability of data (noise) or other factors (Smith and Mason, 1997). Neural network models have a similar problem: the choice of network architecture or topology must be made before training the network on the data. Some researchers suggest that neural networks may not be very robust with respect to changes in this topology. In other words, the performance of a network on the same data should vary given changes to the structure of the network. This "robustness" is not examined in Smith and Mason (1997).

Of the experimental studies in the literature, only one attempts to examine what happens when the hypothesized regression function is different from the "true" function (Smith and Mason, 1997). Other studies appear to be biased in favor of regression models over neural networks because the simple linear functions used to estimate the regression model have the same form as the true function used to generate the data (Markham and Rakes, 1998; Marquez, et al.., 1991).

None of the experimental studies provide a comprehensive comparison of multivariable regression and neural network models in which the only experimental factors are the model formulations. There is a need for a thorough comparative study to determine not only which data analysis technique is more appropriate, but also the conditions under which the cost of refining a particular statistical model is worth the increased accuracy of the model. Additionally, there is no published methodology that assists practitioners in choosing between MLR and ANN when building predictive models.

## Purpose of Study

The purpose of this experimental study is to compare the performance of multiple linear regression and artificial neural networks as data analysis tools in a controlled environment and develop a methodology for guiding practitioners in selecting an appropriate modeling technique. In the experiments, the only variable factors are the *a priori* formulations of the regression function and the neural network topology. The study is designed to test the robustness of regression and neural network models with respect to model accuracy and predictive ability. Robustness is defined as the degree to which a regression function or a neural network can be modified without a significant loss of predictive ability. The independent variable in this study is defined as formulation of the regression and neural network models. The dependent variable is defined as the mean squared error of the regression and neural network models. The null hypothesis being tested is that the root mean squared error (RMSE) for the artificial neural network models is less than the RMSE for the multiple linear regression models.

## Research Questions

Two research questions have been developed to guide this study. These questions distill the research problem and purpose of the study into specific issues to be addressed by the designed experiments. The research questions help define the scope of the research:

- Given identical input vectors, identical training (construction) sample sizes, and identical validation samples, to what degree do variations in model formulation affect the comparative performance of ANN and MLR as measured by root mean squared error (RMSE)?
- How robust are ANN and MLR models to changes in formulation or topology as measured by the variability of the RMSE performance?

# CHAPTER II: REVIEW OF LITERATURE

This research focuses on the intersection of two very broad areas of study: statistical modeling and artificial neural networks. This review of the literature begins with the general area of predictive modeling, gradually narrowing the focus to applications of ANN to statistical modeling problems, and finally to the small, but expanding body of knowledge represented by experimental studies of ANN as a surrogate for MLR to which this research will add.

Figure 1 is a Venn diagram illustration of the representative literature areas. The intersection of all the circles is the focus of this research.

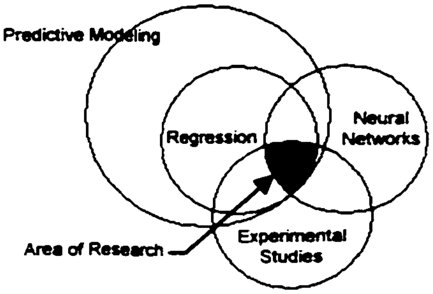


*Figure 1. Area of Research*

## Parametric and Non-Parametric Predictive Modeling

The tools and techniques for the quantitative analysis of data are found in standard applied statistics texts, such as Mendenhall and Sincich (1995) or research-based statistics textbooks such as Dowdy and Wearden (1991) or Kerlinger (1992). Much of this literature covers the foundations of statistical analysis to include both descriptive and inferential statistics. However, these texts also treat extensively the topic of statistical model building, or the creation of an equation that will provide a good fit to a set of data as well as give good predictions of future values of the dependent variable for given values of the independent variables. Regression analysis is only one part of model building, perhaps the least significant part, given the prevalence of powerful statistical analysis software (Berk and Carey, 1995). The actual model construction occurs when one hypothesizes the functional form of the model. According to Mendenhall and Sincich (1995), "if the hypothesized model does not reflect, at least approximately, the true nature of the relationship between the mean response $E(y)$ and the independent variables $x_1, x_2, \ldots, x_k$, the modeling effort will usually be unrewarded" (p. 700).

Traditional statistics and regression modeling is parametric in nature, that is, it is based on probability distributions. The assumption of normality governs the analysis of the residual errors of the regression, for example. The field of non-parametric, or distribution-free statistics opens up the possibility of data analysis in which assumptions regarding an underlying population are not necessary (Gibbons and Chakraborti, 1992; Puri, 1970). Geman (1992) relates the properties of non-parametric model building to artificial neural networks. Non-parametric statistical models have "arbitrary decision boundaries…in the sense that no particular structure, or class of boundaries, is assumed a priori" (p. 1). The link between statistical modeling and neural network modeling is that learning in a neural network "…can be formulated as a (nonlinear) regression problem" (p. 2).

## Artificial Neural Networks

As neural network-based applications have become more commonplace, so the basic literature on neural networks has diverged from the theoretical to the practical. The acknowledged seminal work on backpropagation-based neural networks is Rumelhart and McClelland (1986). However, since this research is application oriented, some of the current general texts on neural networks such as Haykin (1999) and Skapura (1996) provide a very good theoretical basis as well as practical guidance on the construction and application of ANN.

Data presentation and representation in a neural network is critical to a successful application. The previously-cited works also discuss this important area of neural network applications as do Veelenturf (1995) and Lawrence (1991).

Theoretical discussions of the ability of neural networks to serve as universal function approximators are found in Hornik, et al.. (1989), Hartman, et al.. (1990) and White (1989; 1990).

## Applied Neural Network Models

Because of their ability to learn complex, non-linear relationships and generalize this learning to out-of-sample population data, neural networks have been successfully used as prediction models. Artificial neural network prediction models have been used in such diverse areas as economic time series, stock price analysis, academic grading analysis, chemical analysis, meteorology and oceanography.

Much of the application-based literature exploring the use of ANN as surrogates for regression models comes from the field of cost engineering, or more specifically, parametric cost estimating. In parametric cost estimating, physical or performance characteristics of many similar products or processes are collected, along with the cost of the product or process. The object is to use this historical data to build a regression-based predictive model that relates characteristics to cost. The model is then used to predict the cost of a new product or process based on its physical or performance characteristics.

Various application-oriented studies comparing the performance of ANN and MLR are discussed, including several examples from the parametric cost estimating literature.

Paruelo and Tomasel (1997) compared the predictive power of both ANN and MLR in modeling ecosystem attributes. They used 13 years of temperature and precipitation data to empirically derive values for six ecological indices. They found that the ANN generally performed better than regression models based on mean absolute percentage error (MAPE) and coefficient of correlation.

Kwan, et al., (1995) compared both MLR and ANN to previously-derived models for estimating the optimal "tour length" of the traditional traveling salesman problem (TSP). Training data for both MLR and ANN was simulated using variables derived from several configurations of the tour area shape, and the number and location of points in the area. Both MLR and ANN models performed better than the models from the literature, but the neural network models were slightly better than the regression models.

Zeng (1999) discovered that neural network models were a much better prediction tool in social science choice/classification problems than the traditional logit or probit models (which are, typically, linear classifiers). Also using simulated data with a known, "true" function, Zeng (1999) reached the interesting conclusion that the ANN model is statistically indistinguishable from the "true" model.

In a civil engineering application, Owusu-Ababio (1995) used ANN as an alternative to MLR in modeling pavement surface friction as a function of several pavement variables such as regional location and age. The ANN models in this study consistently outperformed the MLR models on both in- and out-of-sample data.

In a pharmacological study focusing on modeling the properties of powders using very limited data, Zolotariov and Anwar (1998) concluded that there was no statistical difference in performance between ANN and MLR models. Their study used a sample size of 33, but a total of 9 independent variables.

Practitioners using ANN as a surrogate for MLR in estimating cost based on historical data have had generally positive results. Bode (1998a,b) collected data for 4 dimensional attributes of 573 different bearings, along with their cost. The resulting network with 4 input nodes, one output node and 6 nodes in one hidden layer (4, 6, 1) performed consistently better than the traditional parametric estimation using regression, even when as few as 20 exemplars were used to train the network.

De-la-Garza and Rouhana (1995) used even fewer data points to train a 3, 4, 1 backpropagation network. Having 16 examples of attribute and cost data for carbon steel pipe, they used only 10 exemplars to train the network and the remaining 6 for testing. Although the data had a strong linear relationship ($R^2 = 0.95$), the neural network provided a 78 percent improvement over a linear regression model. Smith and Mason (1997) take issue with the methodology of de-la-Garza in that all 16 exemplars were used to construct the linear regression models; nevertheless, de-la-Garza concluded that the neural network does represent a significant improvement.

None of the cited cost estimating applications uses more than 4 cost drivers (input neurodes). De-la-Garza and Rouhana (1995) conclude that neural networks can handle a large number of cost drivers when used in cost estimating problems. Bode (1998a,b), however, disagrees, stating that the number of input variables should be limited so as to avoid an overly complex neural network architecture.

### Experiment-Based Literature

Although applications of ANN as an alternative to MLR for predictive modeling have shown promise, these studies are limited because they rely on actual cost, or other modeling data. Research into the nature of neural networks as surrogates to regression necessitates a degree of control over variables in the problem in order to conduct experiments. The ability to generate simulated data based on known functions allows the researcher to control the most important variable in experiment, the mathematical function underlying the data being analyzed.

Several researchers, using simulated data, have experimented with neural networks as alternatives to regression. In most of these studies, the variables of interest were training sample size and noise in the data (represented by the variance of the error term in the underlying function) and their effect on the comparative performance of ANN and regression. Measures of performance were typically mean squared error (MSE) or mean absolute percentage error (MAPE).

Marquez et al. (1991) varied the training sample size, variance of the error term, and the form of the data-generation function. Using linear, logarithmic and reciprocal functions with one independent variable, and sample sizes of 15, 30 and 60 exemplars, the authors compared ANN and regression under a total of 27 different conditions. They used backpropagation to train a network with one hidden layer consisting of 6 neurodes (1, 6, 1). They concluded that ANN outperform regression when sample sizes are small.

Bansal et al. (1993) compared ANN and MLR performance on the same financial data set after simulating the degradation of data. They found that, for this type of data, MLR performed better using R-squared as a performance measure. However, ANN did better when using a payoff criterion tailored to the problem being modeled. They concluded that MLR may have performed better because of a strong linear relationship in the data. They suggested that ANN would likely perform better with non linear relationships in the data, pointing out that specification of a regression model then becomes problematic.

Robinson (1991) conducted a limited experiment with a known function in four independent variables. This function, a second order quadratic with an exponential term, could be considered more representative of the nature of the unknown functions that would be encountered in an application. Both the network and the regression model were “trained” on 100 samples from a set of 200. Only a linear model formulation was used for the regression equation, however. The backpropagation neural network with two hidden layers (4, 15, 7, 1) improved the RMS error over regression by a factor of 10. The author suggests that a neural network cannot discover an underlying relationship from a data sample of fewer than 50 exemplars. This suggestion is questionable, however, given that the author used only a training set of 100 exemplars. Other authors test this notion using factorial experiments and reach different conclusions.

In a very comprehensive experimental study, Smith and Mason (1997) directly compared neural networks to multiple linear regression in determining cost estimating relationships (CER). They examined stability and ease of use as well as performance. A key feature of this study that separates it from previous studies is the attempt to measure the significance of the assumption of the regression model form. The authors compared one neural network (2, 2, 2, 1) to three regression equations representing a best case to worst case estimate of the "known" function. Additionally, they varied training sample size and variance of the error term in the data-generation function. After performing ANOVA on their experimental results, the authors found that CER type (model formulation) was the largest contributor to variability in the data. Size of the training sample contributed relatively little.

Smith and Mason (1997) conclude that an ANN "may be an attractive substitute for regression if… the cost data does not enable fitting a commonly chosen model, or does not allow the analyst to discern the appropriate CER" (p. 156). They also suggest that, as the dimensionality of the input vector increases, the problem is more acute. This implies that ANN should perform much better than regression given a large number of independent variables or cost drivers.

Finally, Markham and Rakes (1998) studied simple linear regression (one independent variable) and neural networks, varying the training sample size and the variance of the error term of the known function. A good deal of pre-optimization was done to determine the "best" neural network to use for the experiments. Once arrived at (1, 2, 1) this network was used for all the experiments. The authors varied sample size from 20 to 500 and variance of the error term from 25 to 400. They concluded (expectedly) that large sample sizes work well for both regression and ANN; however, they favor ANN because of their ability to perform well with large variance levels. When sample size was small, ANN performed better only when variance was high.

Performance of ANN and regression models tended to stabilize and converge rapidly at sample sizes greater than 100.

Table 1 is a summary of some of the salient features of the experimental studies comparing ANN to regression.

| | **Marquez et al. (1991)** | **Robinson (1991)** | **Smith/Mason (1997)** | **Markham/ Rakes (1998)** | **Bansal et al. (1993)** |
|---|---|---|---|---|---|
| **Variables** | Form of underlying function; VAR of error term; sample size | None (non-factorial) | Form of regression model; VAR of error term; sample size; sample bias | VAR of error term; sample size | Data quality (simulated by randomly deviating existing data set) |
| **ANN topology** | 1, 6, 1 | 4, 15, 7, 1 | 2, 2, 2, 1 | 1, 2, 1 | 8, 5, 1 |
| **Conclusions** | ANN perform better w/small sample sizes | ANN perform better when significant non-linearity present in data. ANN cannot perform well when n<50. | ANN perform better when significant non-linearity present in data; also when dimensionality is large. Model formulation significant. | Regression performs better when variance low; ANN when variance high. | MLR performs better if data is linear using $R^2$ as criterion. ANN better w/Payoff criterion. |

*Table 1. Summary of Experimental Studies*

## Conclusions of Literature Review

A review of the literature linking artificial neural networks and multiple linear regression leads to the experimental studies summarized above. All but one of these analyses addresses the effects of sample size and data "noise" on the comparative performance of ANN and regression. After considering the results of the application-oriented literature, it can be concluded that for most types of data, neural networks tend to produce better results than MLR when sample sizes are small. Additionally, neural networks appear to be much better at detecting non-linearities in the data. As Robinson (1991) suggests, traditional regression results might attribute the unexplained relationships in the data to "measurement or environmental noise", when in fact, there are non-linearities in the data that only neural networks can uncover.

### Contribution to the Literature

A gap in the literature on neural networks as a surrogate for regression appears to exist in the area of model formulation. Much has been studied about the effect of sample size and noise on relative performance. However, no comprehensive experimental study has isolated model formulation as a variable for research in this area. Additionally, there has been no published methodology for the combined use of ANN and MLR in predictive modeling. This research should make a necessary contribution to both the theoretical and practical categories of the literature in this area by quantifying the effect of model formulation on the comparative performance of artificial neural networks and regression, and by providing a predictive modeling methodology based on the combined use of ANN and MLR techniques.

# CHAPTER III: RESEARCH METHODOLOGY

The purpose of this research is to explore the robustness of both regression and neural network models with respect to model accuracy and predictive ability. A full-factorial experiment is designed for the comparison of MLR and ANN. Model formulation and its subsequent effect on model performance is studied. To isolate the effects of model formulation on comparative model performance, sample size (construction and validation), dimensionality of the input vector, and variability of the data (as represented by the variance of the error term), are controlled. The backpropagation algorithm is used to train the ANN used in the experiment.

## Sample Size

The construction sample is that portion of the data set used to train, or construct the neural network or upon which the regression is based. In a regression analysis, the construction sample is the data set used to derive the least-square coefficients for the regression model. Validation of the model's generalizability can only be accomplished by testing the model against another sample, drawn from the same population. Although a large data set is helpful when building statistical or ANN models, sometimes data (particularly cost data) may be difficult to come by, forcing the analyst to build a model on a limited number of data points. An assumption of small construction sample size is conservative in that larger data sets can only enhance the quality of the model's output. This study, therefore, assumes a construction sample size of n = 25.

### Size of Input Vector

The term "input vector" is used to describe the number of input neurodes in an ANN. It also represents the number of independent variables in a multivariable regression analysis. In the experimental studies comparing neural networks and regression, some studies use simple linear regression (SLR) with only one independent variable, and some studies use MLR with two independent variables (Marquez, et al., 1991; Markham and Rakes, 1998). However, the typical application-oriented comparison of MLR and ANN used models with three and four independent variables (de la Garza and Rouhana, 1995; Refenes, et al., 1994; Creese and Li, 1995; Bode, 1998; Moselhi and Siquerra, 1998; McKim, 1993).

This research builds on the previous experimental literature by attempting to replicate the conditions found in typical applications of predictive modeling. For this reason the number of independent variables in the study is set at four, providing a more realistic structure for the experimental design of the study.

### Backpropagation Algorithm

The backpropagation algorithm is used to train the neural network models. Backpropagation is a variation of the delta rule, which is a minimum-error learning algorithm (Skapura, 1996; Veelenturf, 1995). Since regression analysis techniques also attempt to fit a minimized error surface to the data, minimum-error algorithms such as backpropagation are appropriate for training neural networks used as surrogates for multiple linear regression. Backpropagation-based ANN have been shown to be robust and easy to implement in a variety of applications, as well as demonstrating the ability to model any continuous, nonlinear function (Haykin, 1999; Eksioglu, 1996).

Table 2 summarizes both the variables under study and the variables to be controlled.

| Variable | Type (study or controlled) | Value |
|---|---|---|
| Formulation of MLR function | Study | Variable |
| Neural network architecture | Study | Variable |
| Construction sample size | Controlled | N = 25 |
| Validation sample size | Controlled | N = 25 |
| Dimensionality of input vector | Controlled | 4 |

*Table 2. Variables in the Study*

## Data Collection

The data for this study is generated using Monte Carlo simulation. The advantage of using simulated data based on a known, multivariable function is that it allows for comparison between the model results and the "true" function. A suitably large population is generated from three separate functions, which has normally distributed error terms with a mean of 0 and a known variance. Introducing an error term into the known function simulates the type of random "noise" found in real-world data. The regression and neural network models built using data samples drawn from this population can then be directly compared to this underlying, known function. Simulated data was also used in previous studies comparing regression and ANN (Marquez et al., 1991; Markham and Rakes, 1998; Smith and Mason, 1997).

There are an infinite number of possible functions that could be used to generate the data for the experiments in this research. The following three functions are chosen:

$$y = x_1^3 + x_1 x_2 + x_3^2 + 20x_4 + \varepsilon(0,10), \tag{1}$$

$$y = \frac{x_1^{0.5} e^{x_2} x_3}{x_4} + \varepsilon(0,6), \tag{2}$$

$$y = 4x_1 + 2.8x_1 x_2 + 0.2x_3 + x_4 + \varepsilon(0,3.5). \tag{3}$$

These functions are chosen because they include four independent variables, representing either variables in a regression model or an input vector for a neural network with a dimensionality of four. They also generate three distinctly different pools of random variates demonstrating varying types of data. Equation 1 is a polynomial function with two nonlinear terms and one interaction term. Equation 2 shows a complex function with both quadratic and exponential relationships between the dependent and independent variables. Finally, equation 3 is a purely linear relationship made slightly more complex with the addition of an interaction term.

| **Independent Variables** | | | | | **Error Terms** | | |
|---|---|---|---|---|---|---|---|
| | $X_1$ | $X_2$ | $X_3$ | $X_4$ | ε (Eq 1) | ε (Eq 2) | ε (Eq 3) |
| Distribution | Uniform | Normal | Uniform | Normal | Normal | Normal | Normal |
| Range | a = 1<br>b = 10 | NA | a = 2<br>b = 8 | NA | NA | NA | NA |
| Mean | 5.5 | 2.8 | 5 | 4 | 0 | 0 | 0 |
| Variance | 6.75 | 0.25 | 3 | 0.04 | 100 | 36 | 12.25 |

*Table 3. Independent Variables and error terms*

Table 3 shows the distribution of each of the independent variables, $x_1$ through $x_4$. The expected range or variance of these independent variables was chosen to keep the dependent variable within a reasonable range across all three functions. Each function has an error term, ε which is normally distributed with a mean of zero and a variance of approximately ten percent of the expected range of the dependent variable.

These three true functions, equations 1, 2, and 3, are used to generate three separate "pools" of 500 exemplars consisting of a dependent variable Y, and four independent variables, $X_1$ through $X_4$. The spreadsheet add-in @Risk is used to generate random variates for these exemplars based on the distributions in Table 3.

Table 4 is a representative listing of 10 exemplars generated using a function similar to equation 1. Each pool consists of 500 exemplars similar in structure to those in Table 4. Although the values of ε are not shown in the table, the effect of this error term is reflected in the value of Y in the exemplar data.

| Y | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|---|
| 1495.82 | 3.62 | 10.16 | 24.03 | 36.66 |
| 1609.44 | 3.51 | 15.53 | 16.38 | 57.29 |
| 1489.35 | 2.80 | 20.50 | 13.28 | 56.49 |
| 2012.00 | 8.78 | 7.61 | 10.18 | 53.45 |
| 1778.09 | 0.31 | 9.08 | 22.87 | 57.29 |
| 2771.06 | 9.77 | 17.13 | 19.23 | 59.45 |
| 1371.24 | 1.12 | 2.37 | 22.29 | 38.50 |
| 2548.84 | 8.08 | 10.96 | 23.08 | 64.14 |
| 2865.44 | 9.55 | 14.86 | 22.82 | 60.35 |
| 1880.79 | 8.34 | 16.14 | 18.62 | 36.66 |

*Table 4. Sample data using a polynomial function*

## Experimental Design

The functions introduced in equations 1, 2, and 3 are used to generate three separate pools of 500 data exemplars. Each exemplar consists of four independent variables and a corresponding dependent variable. Two random samples of size n = 25 are drawn from these pools to be used as construction samples for building the regression models and training the neural networks. Once the models are constructed, an additional random sample of size n = 25 is drawn. The X values from this sample are used to generate the estimated values, $\hat{Y}$. These values are compared to the actual Y value from the sample. The difference is measured in terms of root mean squared error (RMSE):

$$RMSE = \sqrt{\frac{\sum_{i=1}^{25}\left(Y_i - \hat{Y}_i\right)^2}{n}} \qquad (4)$$

where n =25, or the data sample size.

## Experiment Steps

The following steps outline the procedure for conducting the computer experiments for both ANN and MLR models. Figure 2 represents this process in flowchart form:

1) Using Monte Carlo simulation, generate 500 exemplars using the function in equation 1 and the distributions of the random variables $x_1$ through $x_4$.
2) Take three random samples of 25 exemplars each from this pool of 500.
   a) Designate two as training/construction samples.
   b) Designate the remaining sample as a testing/validation sample.
3) Train ANN model 1 with training set 1. Construct MLR model 1 with training set 1.
   a) Use testing/validation set to determine $\hat{Y}$.
   b) Compare with true value, Y.
   c) Determine RMSE.
4) Train ANN model 1 with training set 2. Construct MLR model 2 with training set 2.
   a) Use testing/validation set to determine $\hat{Y}$.
   b) Compare with true value, Y.

  c) Determine RMSE.

5) Average the two RMSE values to produce one RMSE value for ANN model 1 and MLR model 1.
6) Repeat for all remaining ANN and MLR models. There should be one RMSE value for each model.
7) Compare each ANN model with each MLR model using RMSE as a measure of performance (MOP).
8) Repeat steps 1 through 8 for each of the remaining two data-generating functions, equations 2 and 3.

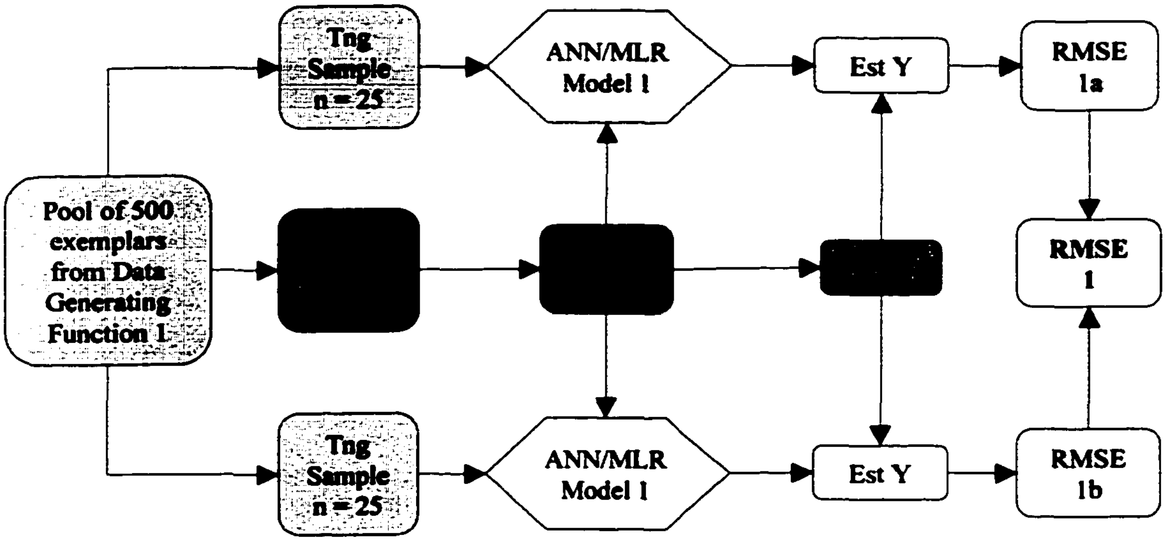


*Figure 2. Experiment flowchart*

## Neural Network Experiment

A factorial experiment is conducted to vary the architecture (model formulation) of the ANN. Three different ANN parameters are varied: the number of processing elements (PE) in the hidden layer, the learning constant value, and the transfer function. The number of PE and the learning constant parameters are set at three levels; the transfer function is set at two levels, for a total of 18 separate ANN models. (The complete factorial experiment matrix can be found at Appendix A). All the models have four input layer neurodes, one for each independent variable, and one output layer neurode for the dependent variable.

The number of processing elements, or neurodes, in the hidden layer(s) has been found to have a significant effect on the ability of ANN to both converge (train to a low level of RMS error) and generalize (Flitman, 1997). However, selecting the number of neurodes and the number of hidden layers is not necessarily a straightforward process. The free parameters within the ANN are the weighted connections between the neurodes. Too many weights (too large a hidden layer) for the data may cause the network to converge quickly, yet not be able to generalize the training to a testing set. Conversely, too few weights for the example data may prevent the network from learning to an acceptable degree of accuracy. Several heuristics exist for determining the number of neurodes in the hidden layer. Flitman (1997) suggests this number can be determined by the following formula:

*Number of hidden neurons = ½ (Inputs + Outputs) + Sqrt(# of training patterns)*

For this research problem, this formula suggests the number of neurodes be limited to approximately 7. Another heuristic, also suggested by Flitman (1997) is simply two times the square root of the sum of the inputs and the outputs, rounded down to the nearest integer. This would result in a hidden layer of 4 neurodes for this experiment. Clearly, it is important to first determine a reasonable value for the number of hidden neurons, and then vary this for purposes of experimentation. For this research, the hidden layers will consist of 3, 6, and 9 neurodes respectively (Table 5).

The type of transfer, or activation, function used in the hidden layer neurodes has an effect on the ability of the network to converge, or minimize the backpropagated error. Typically, a sigmoidal function (Equation 5) is recommended for these networks;

however, other functions such as hyperbolic tangent (Equation 6) have been used successfully (Haykin, 1999; Veelenturf, 1995; Flitman, 1997).

$$y = \frac{1}{1+e^{-x}}, \tag{5}$$

$$y = \frac{e^{x} - e^{x}}{e^{x} + e^{x}}, \tag{6}$$

Both have the characteristic of being monotonically increasing between 0 and 1 (sigmoid) and –1 and 1 (hyperbolic tangent). Since most modern neural network simulation environments offer either sigmoid or hyperbolic tangent (tanh) functions as the default transfer function settings, these two functions are used in the experiments (Table 5).

The learning constant, β, takes values between 0 and 1, and modifies the weight changes between neurodes according to the following equation:

$$\Delta w_{ij} = \beta E f(I) \tag{7}$$

where $\Delta w_{ij}$ is the weight change, E is the error value being propagated back through the neurode, and f(I) is the input to the neurode. A larger value for β makes the individual weight changes larger, which causes the network to train faster. This may or may not have an impact on the quality of training as represented by the RMS error level achieved when the network reaches convergence. Varying the learning constant from 0.3 to 0.9 ensures that a broad range of weight change values is covered.

Table 5 summarizes the various levels of each parameter being modified in the neural networks experiments. The ANN models are developed using NeuroSolutions version 3.02.

| **Parameter** | **Levels** | | |
|---|---|---|---|
| Number of processing elements in hidden layer | 3 | 6 | 9 |
| Learning constant value | 0.3 | 0.6 | 0.9 |
| Transfer function | Sigmoid | Hyperbolic Tangent | N/A |

*Table 5. Neural Network parameters and levels*

## MLR Experiment

For the regression model formulations, a number of different function types are assumed. The objective of using a variety of function types is twofold: 1) to simulate the approach an analyst might take in attempting to fit a regression model to a set of data with an unknown relationship, and 2) to inject variability into the regression estimates of the true functions so the robustness of MLR can be evaluated.

The regression equations are based on the following five types: linear, second and third order polynomials, exponential, and power. Since each model will have one, two, or three interaction terms, there are a total of 15 possible regression models. The functions are listed in Table 6 and the full equations for the regression models can be found at appendix B.

Each of the 15 regression models is built using data sets sampled from the same pools used to construct the ANN models. The estimated values of Y are determined by running the testing data sets drawn from the three data pools through the regression models.

| Model | Function Type | Interaction Terms |
|---|---|---|
| 1 | Linear | 0 |
| 2 | 2nd order polynomial | 0 |
| 3 | 3rd order polynomial | 0 |
| 4 | Exponential | 0 |
| 5 | Power | 0 |
| 6 | Linear | 1 |
| 7 | 2nd order polynomial | 1 |
| 8 | 3rd order polynomial | 1 |
| 9 | Exponential | 1 |
| 10 | Power | 1 |
| 11 | Linear | 2 |
| 12 | 2nd order polynomial | 2 |
| 13 | 3rd order polynomial | 2 |
| 14 | Exponential | 2 |
| 15 | Power | 2 |

*Table 6. Function forms for regression models*

Three of the regression models are functionally identical to the respective data generating functions with the exception of the coefficients (models 4, 6 and 8). These models would, theoretically, be correctly specified, providing a best case scenario for regression. A baseline linear formulation (models 1, 6, and 11) provides the worst case scenario for this study. The best case is a model identical to the true function for which the coefficients must be estimated from the data. Regression models are developed using SPSS for Windows, version 7.5.1.

Normally, when constructing a regression model, a residual analysis is performed to ensure the basic assumptions are met concerning independence, constant variance and normal distribution. Additionally, regression models are normally checked for multicollinearity, or correlations between independent variables. The models in the designed experiments are used directly without this more detailed refinement.

## Data Analysis

For each of the three data pools, every ANN model and MLR model is constructed using the same sample data. Therefore, a one-to-one comparison can be performed using RMSE as a measure of performance. There is a total of 15 x 18 = 270 comparisons per data pool. A matched pair statistical test is used to compare the means of the RMSE differences between ANN and MLR models. The difference is computed using the following equation:

$$\mu_{MLR} - \mu_{ANN} = \mu_d, \tag{8}$$

where $\mu_{ANN}$ and $\mu_{MLR}$ are the RMSE values for the ANN models and MLR models respectively for each pair comparison, and $\mu_d$ is the difference between these values.

If the 95 percent confidence interval for this statistic does not include 0, it can be concluded that one or the other modeling approach is superior depending on whether the sign is negative or positive. If the sign is positive, the ANN models have the lower RMSE values and therefore can be shown to be better predictors than the MLR models. Table 7 shows the software used in constructing the MLR models, constructing and training the ANN models, and analyzing the output of the experiments.

| **Application** | **Vendor** | **Research Use** |
|---|---|---|
| Excel 97 SR-2 | Microsoft Corp. | Spreadsheet software for data management and selecting samples from population. |
| @Risk for Windows, ver. 3.5e | Palisade Corp. | Spreadsheet add-in for Excel. Generates Monte Carlo simulations. Used for generating random variates in the population. |
| SPSS for Windows, ver 7.5.1 (standard) | SPSS, Inc. | Statistical analysis package used for building regression models. |
| NeuroSolutions, ver. 3.02 | NeuroDimensions | Neural networks simulation package for building and training neural network models. |

*Table 7. Software used in research*

# CHAPTER IV: EXPERIMENTAL RESULTS AND DISCUSSION

This chapter presents the experimental results and relates those results to the research questions posed in Chapter I. The first research question asked how variations in model formulation affect the comparative performance of ANN and MLR as measured by RMSE. Each of the 18 ANN models and the 15 MLR models were compared on a one-for-one basis on their ability to accurately estimate three different functional relationships on the basis of artificially generated data. The second research question asked how robust ANN and MLR models were to changes in model formulation or topology.

## Research Question 1: Model Performance

The function in Equations 1 through 3 were used to generate pseudo-populations, or pools, of 500 data exemplars. The experiment steps in Chapter III were followed to train the ANN models and construct the MLR models using the simulated data.

### Function 1 Experiments: ANN Models

The resulting RMSE values for the ANN models trained and tested with the Function 1 data are shown in Table 8. The training and testing samples and the estimated Y values for each of the ANN models are found in Appendix D. These results appear to indicate that the ANN models with the hyperbolic tangent transfer function performed much better than those with the sigmoidal transfer function. A pairwise, two-tailed t-test comparing the nine sigmoid models and the nine hyperbolic tangent models shows a significant difference at an alpha = 0.01 (t-critical = 2.638, and t = 15.08). The hyperbolic tangent models, in addition to having a lower mean RMSE than the sigmoidal models, also had a lower variance, suggesting they are much less sensitive to changes in topology, or model formulation. The variance of the sigmoid models was 1679.00, while the variance of the hyperbolic tangent models was 353.368. The difference is significant at an alpha = 0.05 (F-critical = 3.438, and F = 4.728).

| Model | Processing Elements | Learning Coefficient | Transfer Function | Average RMSE |
|---|---|---|---|---|
| 1 | 3 | 0.3 | Sigmoid | 109.10 |
| [illegible] | [illegible] | 0.3 | [illegible] | [illegible] |
| 3 | 9 | 0.3 | Sigmoid | 127.19 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 5 | 6 | 0.6 | Sigmoid | 161.15 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 7 | 3 | 0.9 | Sigmoid | 241.28 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 9 | 9 | 0.9 | Sigmoid | 89.06 |
| 10 | [illegible] | [illegible] | [illegible] | [illegible] |
| 11 | 6 | 0.3 | TanH | 36.61 |
| 12 | 9 | 0.3 | TanH | [illegible] |
| 13 | 3 | 0.6 | TanH | 97.56 |
| [illegible] | [illegible] | 0.6 | TanH | [illegible] |
| 15 | 9 | 0.6 | TanH | 38.68 |
| 16 | [illegible] | 0.9 | TanH | [illegible] |
| 17 | 6 | 0.9 | TanH | 76.08 |
| 18 | 9 | 0.9 | TanH | [illegible] |

*Table 8. Function 1 ANN Models*

## Function 1 Experiments: MLR Models

The resulting RMSE values for the MLR models constructed and tested with the Function 1 data are shown in Table 9. The construction and testing samples and the estimated Y values for each of the MLR models are found in Appendix E. The mean RMSE value for all 15 models was 69.24 with a variance of 2580.65.

| Model | Function Type | Interaction Terms | Avg RMSE |
|---|---|---|---|
| 1 | Linear | 0 | 108.73 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 3 | Poly-3 | 0 | 13.56 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 5 | Power | 0 | 177.22 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 7 | Poly-2 | 1 | 53.36 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 9 | Exp | 1 | 29.25 |
| 10 | [illegible] | [illegible] | [illegible] |
| 11 | Linear | 2 | 90.40 |
| 12 | [illegible] | [illegible] | [illegible] |
| 13 | Poly-3 | 2 | 9.77 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 15 | Power | 2 | 105.31 |

*Table 9. Function 1 MLR Models*

## Performance Comparison

A paired t-test was performed comparing each of the 18 ANN models with each of the 15 MLR models for a total of 270 pairs with a hypothesized mean difference of 0. The t-statistic based on the overall paired differences was –7.546, which indicates a significant difference in performance between the ANN models and the MLR models at an alpha of 0.01 (t-critical = -2.576). The 99 percent confidence interval for the mean difference between the two model types was entirely negative, indicating that the MLR models performed better overall in estimating the data generated by Function 1. Table 10 is a summary of the performance comparison and clearly shows the overall performance of the MLR models is better than that of the ANN models. Even a direct comparison of just the linear formulations of the MLR models showed no significant difference in performance from the ANN models. However, it is the ANN models with the sigmoidal transfer functions that bring down the overall performance of the neural networks. A comparison of the hyperbolic tangent ANN models and the MLR models shows no significant difference in performance at an alpha of 0.05, indicating that the best ANN models do not outperform the MLR models for n = 25.

The lower variance for the hyperbolic tangent ANN models suggests they are more robust with respect to changes in the other parameters (number of processing elements and learning constant) than MLR models. The difference is significant at the 1 percent level (F-critical = 3.237, F = 6.540).

| | ANN Models | | | MLR Models | |
|---|---|---|---|---|---|
| | [illegible] | Tan H | Sigmoid | [illegible] | Linear |
| Mean | [illegible] | 63.467 | 139.05 | [illegible] | 101.943 |
| Variance | [illegible] | 353.368 | 1679.00 | [illegible] | 68.632 |

*Table 10. Performance Comparison, Function 1*

The same 18 ANN models and 15 MLR models were then used to estimate Function 2 from the data generated by Equation 2.

## Function 2 Experiments: ANN Models

The resulting RMSE values for the ANN models trained and tested with the Function 2 data are shown in Table 11. As with the results from Function 1, the models with the hyperbolic tangent transfer function performed significantly better than those with the sigmoid transfer function at an alpha = 0.01 (t-critical = 2.638 and a t-statistic of 5.962). Again, the hyperbolic tangent models had a lower variance than the sigmoid models, indicating a higher level of robustness. The variance of the sigmoid models was 94.368 while the variance of the hyperbolic tangent models was 19.120. The difference is significant at the 5 percent level (F-critical = 3.438, F = 4.935).

| Model | Processing Elements | Learning Coefficient | Transfer Function | Average RMSE |
|---|---|---|---|---|
| 1 | 3 | 0.3 | Sigmoid | 23.97 |
| 2 | 6 | 0.3 | Sigmoid | [illegible] |
| 3 | 9 | 0.3 | Sigmoid | 23.14 |
| 4 | 3 | 0.6 | Sigmoid | [illegible] |
| 5 | 6 | 0.6 | Sigmoid | 19.12 |
| 6 | 9 | 0.6 | Sigmoid | [illegible] |
| 7 | 3 | 0.9 | Sigmoid | 20.42 |
| 8 | 6 | 0.9 | Sigmoid | [illegible] |
| 9 | 9 | 0.9 | Sigmoid | 19.90 |
| 10 | 3 | 0.3 | TanH | [illegible] |
| 11 | 6 | 0.3 | TanH | 14.47 |
| 12 | 9 | 0.3 | TanH | [illegible] |
| 13 | 3 | 0.6 | TanH | 20.85 |
| 14 | 6 | 0.6 | TanH | [illegible] |
| 15 | 9 | 0.6 | TanH | 12.49 |
| 16 | 3 | 0.9 | TanH | 14.72 |
| 17 | 6 | 0.9 | TanH | 10.53 |
| 18 | 9 | 0.9 | TanH | 17.36 |

*Table 11. Function 2 ANN Models*

## Function 2 Experiments: MLR Models

The RMSE values for the MLR models constructed and tested with the Function 2 data are shown in Table 12. The mean RMSE value for all 15 models was 15.18 with a variance of 26.49. Function 2 had an exponential term as well as a square root term and the power and exponential model formulations appeared to perform the best on these data.

| Model | Function Type | Interaction Terms | Average RMSE |
|---|---|---|---|
| 1 | Linear | 0 | 12.61 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 3 | Poly-3 | 0 | 20.00 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 5 | Power | 0 | 7.56 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 7 | Poly-2 | 1 | 17.17 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 9 | Exp | 1 | 11.97 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 11 | Linear | 2 | 10.89 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 13 | Poly-3 | 2 | 27.53 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 15 | Power | 2 | 16.91 |

*Table 12. Function 2 MLR Models*

## Performance Comparison

As with Function 1, a paired t-test was performed comparing the results of each of the 18 ANN models with those of each of the 15 MLR models, for a total of 270 pairs. The hypothesized mean difference was 0. The t-statistic based on the overall paired differences was –6.629, indicating a significant difference in performance between the ANN and the MLR models at an alpha of 0.01 (t-critical = -2.594). The 99 percent confidence interval for the mean difference between the two model types was again entirely negative, indicating the MLR models performed better overall in estimating Function 2 based on the generated data. Table 13 summarizes the performance comparison and shows the overall performance of the MLR models as superior to that of the ANN models. Overall variance was significantly lower for the MLR models at the 5 percent level of significance (F-critical = 0.412, F = 0.390). A simple linear formulation of the MLR models performed better than the ANN models overall. In addition, the linear MLR formulations performed better than the best ANN models, which were the hyperbolic tangent models. The variance of the hyperbolic tangent ANN models was not statistically different than the overall variance of the MLR models, suggesting that for this function type, the MLR models were more robust overall than the ANN models.

| | ANN Models | | | MLR Models | |
|---|---|---|---|---|---|
| | [illegible] | Tan H | Sigmoid | [illegible] | Linear |
| Mean | [illegible] | 15.594 | 22.651 | [illegible] | 12.113 |
| Variance | [illegible] | 19.120 | 94.368 | [illegible] | 0.792 |

*Table 13. Performance Comparison, Function 2*

The function in Equation 3 was used to generate the data exemplars for the third set of experiments comparing the 18 ANN models with the 15 MLR models. It was a simple linear function with one interaction term, or cross product.

## Function 3 Experiments: ANN Models

The resulting RMSE values for the ANN models trained and tested with the Function 3 data are shown in Table 14. As with the previous two data sets, these results appear to indicate that the ANN models with the hyperbolic tangent transfer function performed much better than those with the sigmoid transfer function. A two-tailed paired t-test comparing the nine hyperbolic tangent models and the nine sigmoid models shows a significant difference in performance at an alpha of 0.01 (t-critical = 2.638, and t-statistic = 14.183). However, the variance of the hyperbolic tangent models was not statistically different than that of the sigmoid models at an alpha = 0.05 (F-critical = 3.438, F = 2.151).

| Model | Processing Elements | Learning Coefficient | Transfer Function | Average RMSE |
|---|---|---|---|---|
| 1 | 3 | 0.3 | Sigmoid | 12.78 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 3 | 9 | 0.3 | Sigmoid | 12.23 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 5 | 6 | 0.6 | Sigmoid | 13.55 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 7 | 3 | 0.9 | Sigmoid | 12.03 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 9 | 9 | 0.9 | Sigmoid | 10.60 |
| 10 | [illegible] | [illegible] | [illegible] | [illegible] |
| 11 | 6 | 0.3 | TanH | 7.50 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 13 | 3 | 0.6 | TanH | 10.15 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 15 | 9 | 0.6 | TanH | 5.63 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 17 | 6 | 0.9 | TanH | 6.87 |
| 18 | 9 | 0.9 | TanH | [illegible] |

*Table 14. Function 3 ANN Models*

## Function 3 Experiments: MLR Models

The resulting RMSE values for the MLR models constructed and tested with the Function 3 data are shown in Table 15. The mean RMSE value for all 15 models was 7.55 with a variance of 10.71. As expected, because of the linear data-generating function, the linear formulations performed slightly better than the other MLR models. However, it is interesting to note that MLR model 6, the exact specification of the underlying function, did not perform as well as either MLR Model 1 or Model 11, with zero and 2 interaction terms, respectively.

| Model | Function Type | Interaction Terms | Average RMSE |
|---|---|---|---|
| 1 | Linear | 0 | 4.22 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 3 | Poly-3 | 0 | 11.36 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 5 | Power | 0 | 3.85 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 7 | Poly-2 | 1 | 11.55 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 9 | Exp | 1 | 11.82 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 11 | Linear | 2 | 3.39 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 13 | Poly-3 | 2 | 4.77 |
| [illegible] | [illegible] | [illegible] | [illegible] |
| 15 | Power | 2 | 7.21 |

*Table 15. Function 3 MLR Models*

## Performance Comparison

As with the previous two functions, the 18 ANN models and the 15 MLR models were compared on a one-for-one basis using the training and testing data generated by Function 3. A paired t-test was performed on the 270 pairs of RMSE results with a hypothesized mean difference of zero. The t-statistic based on the overall paired differences was –10.829, which indicates a significant difference in performance between the ANN models and the MLR models at an alpha of 0.01 (t-critical = -2.594). The 99 percent confidence interval for the mean difference between the two model types was, again, entirely negative, indicating the MLR models performed better overall in estimating the Function 3 based on the simulated data. Table 16 is a summary of the performance comparison and shows that the overall performance of the MLR models based on mean RMSE values is better than that of the ANN models. The variances are not statistically different. Eliminating the sigmoid-based ANN models reduces both the mean RMSE as well as the variance. However, there is no statistical difference (at the 5 percent level of significance) between the performance of the hyperbolic tangent-based ANN models and the overall MLR models. The linear models performed better than the best ANN models, probably because the underlying functional relationship was based on a first order linear function. The variance of the hyperbolic tangent models is lower than the overall variance of the MLR models, however the ratio is only statistically significant at the 10 percent level, (F-critical = 2.475, F = 2.979) suggesting a slightly higher degree of robustness with respect to model formulation.

| | ANN Models | | | MLR Models | |
|---|---|---|---|---|---|
| | [illegible] | Tan H | Sigmoid | Overall | Linear |
| Mean | [illegible] | 8.108 | 13.306 | [illegible] | 4.265 |
| Variance | [illegible] | 3.435 | 7.426 | [illegible] | 0.562 |

*Table 16. Performance Comparison, Function 3*

### Summary of ANN and MLR Comparison Results

Table 17 summarizes the statistical comparison between the 18 ANN models and the 15 MLR in their ability to estimate the three test functions based on the simulated data. The overall comparison of means across the three data sets shows the MLR models performing better than the neural networks. There was no statistical difference in the model variances except for Function 2, in which the MLR models had a lower variance.

| | **Lowest Mean RMSE** | | **Lowest Variance** | |
|---|---|---|---|---|
| | Overall | Eliminating Sigmoid Models | Overall | Eliminating Sigmoid Models |
| **Function 1** | MLR | [illegible] | No Difference | [illegible] |
| **Function 2** | MLR | [illegible] | MLR | [illegible] |
| **Function 3** | MLR | [illegible] | No Difference | [illegible] |

*Table 17. Summary of ANN and MLR comparison results*

However, it is apparent that, for all three data sets, there is improvement in the performance of the ANN models when those with sigmoid transfer functions are eliminated from the comparison. This may be an indication that the hyperbolic tangent transfer function is more suitable for these types of data analysis problems. After eliminating the sigmoid-based ANN models from the comparison, there is no statistical difference in mean RMSE performance between the ANN and MLR models. In addition, the hyperbolic tangent-based ANN models have a generally lower variance than the MLR models. This lower variance is statistically significant for the Function 1 data and suggests that neural network models may be less sensitive to changes in model formulation and therefore, more robust.

### Sample Size 50 Excursion: Performance Comparison

The literature suggests that when sample size is small and data variance fairly high, neural network models should perform better than multivariate linear regression models (Markham and Rakes, 1998). The fact that, across all three data sets, there was no significant difference in performance between the ANN (hyperbolic tangent) and MLR models for $n = 25$ may suggest that the error terms used in the data-generating functions (equations 1, 2, and 3) did not contribute a great deal of noise to the data relative to the sample size.

An excursion experiment was performed in which the same 18 ANN models and 15 MLR models were compared on the Function 1 data set but with training and testing sample sizes of $n = 50$. The purpose of this excursion was to learn how an increase in sample size without changing the noise level would affect the comparative performance of these models.

## ANN Models

Table 18 shows the change in performance of the 18 ANN models for Function 1 when the sample size is increased to 50. On a model-for-model basis, there was an average overall improvement of 17.09 percent. A paired t-test between the two sets of results shows that this improvement is statistically significant at the 5 percent significance level (p = 0.022).

| Model | Processing Elements | Learning Coefficient | Transfer Function | Avg RMSE (n = 25) | Avg RMSE (n = 50) | Percent Improvement |
|---|---|---|---|---|---|---|
| 1 | 3 | 0.3 | Sigmoid | 109.10 | 137.42 | -25.96 |
| 2 | [illegible] | 0.3 | [illegible] | [illegible] | 104.91 | [illegible] |
| 3 | 9 | 0.3 | Sigmoid | 127.19 | 114.85 | 9.71 |
| 4 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 5 | 6 | 0.6 | Sigmoid | 161.15 | 113.72 | 29.43 |
| 6 | [illegible] | 0.6 | [illegible] | [illegible] | [illegible] | [illegible] |
| 7 | 3 | 0.9 | Sigmoid | 241.28 | 105.95 | 56.09 |
| 8 | [illegible] | 0.9 | [illegible] | [illegible] | 123.40 | [illegible] |
| 9 | 9 | 0.9 | Sigmoid | 89.06 | 105.06 | -17.97 |
| 10 | [illegible] | [illegible] | [illegible] | [illegible] | 36.02 | [illegible] |
| 11 | 6 | 0.3 | TanH | 36.61 | 36.27 | 0.92 |
| 12 | [illegible] | [illegible] | [illegible] | [illegible] | 41.05 | [illegible] |
| 13 | 3 | 0.6 | TanH | 97.56 | 39.56 | 59.45 |
| 14 | [illegible] | 0.6 | TanH | [illegible] | 47.16 | [illegible] |
| 15 | 9 | 0.6 | TanH | 38.68 | 45.88 | -18.62 |
| 16 | [illegible] | 0.9 | TanH | [illegible] | 50.84 | [illegible] |
| 17 | 6 | 0.9 | TanH | 76.08 | 48.02 | 36.88 |
| 18 | [illegible] | 0.9 | TanH | [illegible] | 61.46 | [illegible] |
|  |  |  |  |  | **Average Improvement:** | **17.09 %** |

*Table 18. Percent change in ANN model performance with n = 50*

The overall variance of the model results improves as well when sample size is increased. The variance of the RMSE performance for the 18 ANN models trained and tested on sample sizes of 25 was 2,575.85. Increasing the sample size to 50 for the same

18 models reduced the variance to 1,403.31, a reduction of almost 50 percent. However, this variance reduction was not statistically significant at the 5 percent level (p = 0.11).

## MLR Models

Table 19 shows the change in performance of the 15 MLR models when the sample size was increased from 25 to 50 for both construction and validation samples. Although the overall average performance of the MLR models declined when compared on a one-for-one basis, a paired t-test indicates no significant difference in performance at the 5 percent level (p = 0.866). Likewise, there is no statistically significant difference in variance (p = 0.288). Essentially, increasing the sample size did nothing to improve the performance of the regression models.

| Model | Function Type | Interaction Terms | Average RMSE (n = 25) | Average RMSE (n = 50) | Percent Improvement |
|---|---|---|---|---|---|
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 2 | Poly-2 | 0 | 49.98 | 39.05 | 21.88 |
| 3 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 4 | Exp | 0 | 146.03 | 100.95 | 30.87 |
| 5 | [illegible] | 0 | [illegible] | [illegible] | [illegible] |
| 6 | Linear | 1 | 106.70 | 108.57 | -1.75 |
| 7 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 8 | Poly-3 | 1 | 11.95 | 11.00 | 7.94 |
| 9 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 10 | Power | 1 | 49.49 | 106.91 | -116.02 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 12 | Poly-2 | 2 | 44.56 | 41.77 | 6.27 |
| 13 | [illegible] | [illegible] | [illegible] | 12.02 | [illegible] |
| 14 | Exp | 2 | 42.39 | 51.13 | -20.62 |
| 15 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
|  |  |  |  | Average Improvement: | -6.17 |

*Table 19. Percent change in MLR model performance with n = 50*

## Performance Comparison

There is still a significant difference in overall mean performance between the 18 ANN models and the 15 MLR models: The regression models still perform better based on mean RMSE values; however, there is still no statistically significant difference in variance. As with the smalier sample sizes, the hyperbolic tangent ANN models performed significantly better than the sigmoid models, suggesting that transfer function type is not an appropriate model parameter for adjustment in regression problems using neural network models. When the hyperbolic tangent ANN models are compared to the MLR models, there is an improvement in performance by the ANN models which is significantly different from that of the MLR models at the 1 percent level. Table 20 summarizes the performance comparison between the ANN and MLR models for sample size 50. Eliminating the transfer function as a model parameter also improves the variance of the model results. The difference is highly significant (p = 0.0000297). In the following sections, an extensive analysis of variance (ANOVA) is performed to determine which model parameters (experimental factors) contribute the most variation in model performance.

| | ANN Models | | | MLR Models | |
|---|---|---|---|---|---|
| | [illegible] | Tan H | Sigmoid | [illegible] | Linear |
| Mean | [illegible] | 45.029 | 115.623 | [illegible] | 105.592 |
| Variance | [illegible] | 60.501 | 100.105 | [illegible] | 46.703 |

*Table 20. Performance comparison, Function 1 and n = 50*

## Research Question 2: Model Robustness

The remaining research question related to the robustness of MLR and ANN models. It would be desirable for a predictive modeling technique to be robust with respect to changes in model parameters. In the case of MLR models, the predicted outcome should not only be as accurate as possible, it should be relatively insensitive to the bias between the "true" functional relationship between the independent and dependent variables and the hypothesized functional relationship. Such robustness is useful when the underlying functional relationship is not easily discerned from a study of the data. For ANN models, predicted results should be insensitive to changes in the magnitude of learning coefficients or numbers of processing elements in the hidden layer.

The variability of the RMSE results from model to model is a measure of the robustness of the modeling technique. Low variability indicates a robust approach, while high variability indicates a correspondingly high degree of sensitivity of model performance to changes in model parameters, and hence, a non-robust approach.

For each of the three data sets, the variance of the experimental results of the 18 ANN models and the 15 MLR models was studied using the analysis of variance, or ANOVA. Analysis of variance can provide information about which experimental factors (model parameters) contribute the most to the variability of the results.

The approach used in this study is that suggested by Mendenhall and Sincich (1995) in their chapter on designed experiments when the experimental factors are qualitative. The authors suggest building a linear model of the factorial design of experiments, taking into consideration both the main effects of each factor as well as the interaction effects. Dummy variables can be used if some or all of the factors are qualitative. A multiple linear regression of this linear model is performed using SPSS with the resulting ANOVA output.

| | | | | Main Effects | | | | | Interaction Effects | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| PE | LC | TF | y | x1 | x2 | x3 | x4 | x5 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | x1x3x5 | [illegible] | [illegible] | [illegible] |
| 3 | 0.3 | Sig | 109.10 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 6 | 0.3 | Sig | 129.37 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 9 | 0.3 | Sig | 127.19 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0.6 | Sig | 124.46 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 6 | 0.6 | Sig | 161.15 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 9 | 0.6 | Sig | 127.61 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 3 | 0.9 | Sig | 241.28 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0.9 | Sig | 142.24 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0.9 | Sig | 89.06 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0.3 | TanH | 51.21 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0.3 | TanH | 36.61 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0.3 | TanH | 75.03 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0.6 | TanH | 97.56 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0.6 | TanH | 60.37 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0.6 | TanH | 38.68 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0.9 | TanH | 58.68 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0.9 | TanH | 76.08 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0.9 | TanH | 76.99 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Dummy Variables:**
x1 = 1 if 3 PE, 0 if not
x2 = 1 if 6 PE, 0 if not
x3 = 1 if LC is .3, 0 if not
x4 = 1 if LC is .6, 0 if not
x5 = 1 if Sigmoid, 0 if not

*Table 21. Linear model for Function 1 ANN results*

## Function 1 Robustness Analysis: ANN Results

Table 21 details the linear model for the experimental results from the Function 1 data for the 18 ANN models. The binary dummy variables x1 through x5 describe the relationship of the three factors, number of processing elements in the hidden layer, learning coefficient, and transfer function type, to the resulting RMSE. The variables x1 and x2 correspond to number of processing elements, x3 and x4 correspond to learning coefficient value, and x5 corresponds to transfer function type. The linear model takes the form:

$$y = \beta_0 + \beta x1 + \beta_2 x2 + \beta_3 x3 + \beta_4 x4 + \beta_5 x5 + \beta_6 x1x3 + \beta_7 x1x4 + \beta_8 x1x5 + \beta_9 x2x3$$
$$\beta_{10} x2x4 + \beta_{11} x2x5 + \beta_{12} x3x5 + \beta_{13} x4x5 + \beta_{14} x1x3x5 + \beta_{15} x1x4x5 + \beta_{16} x2x3x5 + \beta_{17} x2x4x5 \qquad (9)$$

where the coefficients $\beta_1$ through $\beta_5$ describe the main effects of the factors and $\beta_6$ through $\beta_{17}$ describe the interaction effects.

Table 22 shows the SPSS ANOVA output with regression results of the model in Equation 9. The criterion for inclusion in the stepwise regression process was a probability of an F-statistic of less than or equal to 0.15. Only one linear model was significant with the variable x5, representing the factor transfer function type, as the only predictor. This result is consistent with the finding that there is a significant improvement in the performance of the ANN models when the transfer function is changed from sigmoid to hyperbolic tangent. It is clear from the ANOVA that transfer function should not have been included as an experimental factor. Its overwhelming contribution to the performance of the models suggests that the clear choice for ANN models used as surrogates for MLR is a hyperbolic tangent transfer function.

**ANOVA**[b]

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 25707.979 | 1 | 25707.979 | 22.749 | .000[a] |
| | Residual | 18081.212 | 16 | 1130.076 | | |
| | Total | 43789.191 | 17 | | | |

[a] Predictors: (Constant), X5

[b] Dependent Variable: Y

*Table 22. ANOVA of Function 1 ANN linear model*

**ANOVA**[g]

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 1307.824 | 1 | 1307.824 | 4.951 | .061[a] |
| | Residual | 1848.934 | 7 | 264.133 | | |
| | Total | 3156.758 | 8 | | | |
| 2 | Regression | 1891.269 | 2 | 945.634 | 4.483 | .064[b] |
| | Residual | 1265.489 | 6 | 210.915 | | |
| | Total | 3156.758 | 8 | | | |
| 3 | Regression | 2357.831 | 3 | 785.944 | 4.919 | .059[c] |
| | Residual | 798.927 | 5 | 159.785 | | |
| | Total | 3156.758 | 8 | | | |
| 4 | Regression | 2891.616 | 4 | 722.904 | 10.906 | .020[d] |
| | Residual | 265.142 | 4 | 66.285 | | |
| | Total | 3156.758 | 8 | | | |
| 5 | Regression | 3126.928 | 5 | 625.386 | 62.894 | .003[e] |
| | Residual | 29.830 | 3 | 9.943 | | |
| | Total | 3156.758 | 8 | | | |
| 6 | Regression | 3154.850 | 6 | 525.808 | 551.078 | .002[f] |
| | Residual | 1.908 | 2 | .954 | | |
| | Total | 3156.758 | 8 | | | |

a. Predictors: (Constant), X1X4

b. Predictors: (Constant), X1X4, X2X3

c. Predictors: (Constant), X1X4, X2X3, X4

d. Predictors: (Constant), X1X4, X2X3, X4, X1

e. Predictors: (Constant), X1X4, X2X3, X4, X1, X2X4

f. Predictors: (Constant), X1X4, X2X3, X4, X1, X2X4, X1X3

g. Dependent Variable: Y

*Table 23. ANOVA of Function 1 ANN linear model without transfer function factor*

To determine how sensitive the ANN models are to changes in the remaining factors (number of processing elements and learning coefficient) the linear model (Equation 9) was altered to eliminate the variable x5 from the main effects and the interaction effects. Table 23 contains the SPSS ANOVA output with the results of the altered linear model. The F-statistics are less significant (still significant at the 10 percent level) and the criterion for inclusion in the stepwise regression process had to be raised to a probability of F of less than or equal to 0.15 to capture several variations of the linear model. It is notable that all the resulting linear regression models contain one or more interaction terms. From Table 24 it can be seen that interactions between the factors account for almost half of the variability of the RMSE values. These results suggest that ANN models are more tightly knit and less sensitive to changes in individual model parameters. In other words, the ANN models are more robust than the MLR models.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .644[a] | .414 | .331 | 16.2522 |
| 2 | .774[b] | .599 | .465 | 14.5229 |
| 3 | .864[c] | .747 | .595 | 12.6406 |
| 4 | .957[d] | .916 | .832 | 8.1416 |
| 5 | .995[e] | .991 | .975 | 3.1533 |
| 6 | 1.000[f] | .999 | .998 | .9768 |

[a] Predictors: (Constant), X1X4

[b] Predictors: (Constant), X1X4, X2X3

[c] Predictors: (Constant), X1X4, X2X3, X4

[d] Predictors: (Constant), X1X4, X2X3, X4, X1

[e] Predictors: (Constant), X1X4, X2X3, X4, X1, X2X4

[f] Predictors: (Constant), X1X4, X2X3, X4, X1, X2X4, X1X3

*Table 24. R-Squared values for Function 1 ANN linear models*

## Function 1 Robustness Analysis: MLR Results

Table 25 details the linear model for the experimental results from the Function 1 data. The binary "dummy" variables x1 through x6 describe the relationship of the two factors, function type and number of interaction terms, to RMSE. The variables x1 through x4 relate to function type, while x5 and x6 relate to number of interaction terms. The actual linear model takes the form:

$$y = \beta_0 + \beta x1 + \beta_2 x2 + \beta_3 x3 + \beta_4 x4 + \beta_5 x5 + \beta_6 x6 +$$
$$\beta_7 x1x5 + \beta_8 x1x6 + \beta_9 x2x5 + \beta_{10} x2x6 + \beta_{11} x3x5 + \beta_{12} x3x6 + \beta_{13} x4x5 + \beta_{14} x4x6 \tag{10}$$

where the coefficients $\beta_1$ through $\beta_6$ describe the main effects of the factors and $\beta_7$ through $\beta_{14}$ describe the interaction effects.

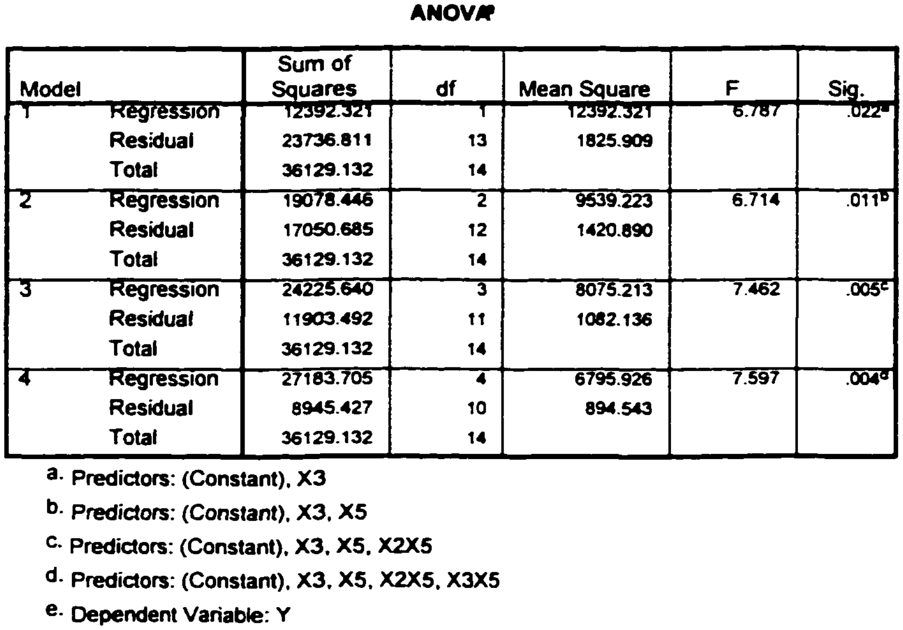
|  |  |  | Main Effects |  |  |  |  |  | Interaction Effects |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Function Type | Interaction Terms | y | x1 | x2 | x3 | x4 | x5 | x6 | x1x5 | x1x6 | x2x5 | x2x6 | x3x5 | x3x6 | x4x5 | x4x6 |
| Linear | 0 | 108.73 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Poly-2 | 0 | 49.98 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Poly-3 | 0 | 13.56 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| Exp | 0 | 146.03 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Power | 0 | 177.22 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Linear | 1 | 106.70 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Poly-2 | 1 | 53.36 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| Poly-3 | 1 | 11.95 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Exp | 1 | 29.25 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Power | 1 | 49.49 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Linear | 2 | 90.40 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Poly-2 | 2 | 44.56 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Poly-3 | 2 | 9.77 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Exp | 2 | 42.39 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Power | 2 | 105.31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Dummy Variables:

X1 = 1 if Linear, 0 if not
X2 = 1 if Polynomial-2, 0 if not
X3 = 1 if Polynomial-3, 0 if not
X4 = 1 if Exponential, 0 if not
X5 = 1 if 0 interaction terms, 0 if not
X6 = 1 if 1 interaction term, 0 if not

*Table 25. Linear model for Function 1 MLR results*

Table 26 contains the SPSS ANOVA output of the results of the regression of the model in Equation 10. The criterion for inclusion in the stepwise regression process was a probability of an F-statistic of less than or equal to 0.100. All the variables are significant at the 5 percent level. The ANOVA results show that the most significant variables in the linear model are x3 and x5, which relate directly to the factor function type. The fact that the main effects in this model predominate suggests that MLR models are highly sensitive to the nature of the hypothesized function and therefore, not very robust with respect to this model parameter.



**ANOVA**[e]

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 12392.321 | 1 | 12392.321 | 6.787 | .022[a] |
| | Residual | 23736.811 | 13 | 1825.909 | | |
| | Total | 36129.132 | 14 | | | |
| 2 | Regression | 19078.446 | 2 | 9539.223 | 6.714 | .011[b] |
| | Residual | 17050.685 | 12 | 1420.890 | | |
| | Total | 36129.132 | 14 | | | |
| 3 | Regression | 24225.640 | 3 | 8075.213 | 7.462 | .005[c] |
| | Residual | 11903.492 | 11 | 1082.136 | | |
| | Total | 36129.132 | 14 | | | |
| 4 | Regression | 27183.705 | 4 | 6795.926 | 7.597 | .004[d] |
| | Residual | 8945.427 | 10 | 894.543 | | |
| | Total | 36129.132 | 14 | | | |

a. Predictors: (Constant), X3

b. Predictors: (Constant), X3, X5

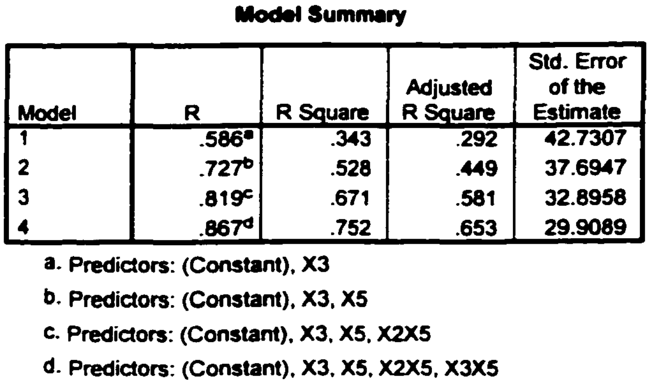
c. Predictors: (Constant), X3, X5, X2X5

d. Predictors: (Constant), X3, X5, X2X5, X3X5

e. Dependent Variable: Y

*Table 26. ANOVA of Function 1 linear model: MLR results*

Table 27 summarizes the adjusted R-squared values for four possible linear models of the Function 1 results. The variable x3 (Poly-3) contributes almost 30 percent of the variability of the model. Main effects in general (x3 and x5) contribute 45 percent or almost half of the variability in this linear model. Interaction effects do not enter the regression process until model 3.



**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .586[a] | .343 | .292 | 42.7307 |
| 2 | .727[b] | .528 | .449 | 37.6947 |
| 3 | .819[c] | .671 | .581 | 32.8958 |
| 4 | .867[d] | .752 | .653 | 29.9089 |

a. Predictors: (Constant), X3

b. Predictors: (Constant), X3, X5

c. Predictors: (Constant), X3, X5, X2X5

d. Predictors: (Constant), X3, X5, X2X5, X3X5

*Table 27. R-Squared values for Function 1 linear model*

The signs for the coefficients are negative for all but x5 (number of interaction terms) indicating that, in this case, it is either function type or the interaction of function type and number of cross terms that are associated with lower RMSE values.

## Function 2 Robustness Analysis: ANN Results

Table 28 details the linear model for the experimental results from the Function 2 data for the 18 ANN models. The binary dummy variables x1 and x2 correspond to the number of processing elements, x3 and x4 to the level of the learning coefficient, and x5 to the transfer function type. The linear model is identical to Equation 9 where the coefficients describe both the main effects and the interactions of the three experimental factors.

| | | | | Main Effects | | | | |
|---|---|---|---|---|---|---|---|---|
| PE | LC | TF | y | x1 | x2 | x3 | x4 | x5 |
| 3 | 0.3 | Sigmoid | 23.97 | 1 | 0 | 1 | 0 | 1 |
| 6 | 0.3 | Sigmoid | 14.49 | 0 | 1 | 1 | 0 | 1 |
| 9 | 0.3 | Sigmoid | 23.14 | 0 | 0 | 1 | 0 | 1 |
| 3 | 0.6 | Sigmoid | 48.46 | 1 | 0 | 0 | 1 | 1 |
| 6 | 0.6 | Sigmoid | 19.12 | 0 | 1 | 0 | 1 | 1 |
| 9 | 0.6 | Sigmoid | 14.08 | 0 | 0 | 0 | 1 | 1 |
| 3 | 0.9 | Sigmoid | 20.42 | 1 | 0 | 0 | 0 | 1 |
| 6 | 0.9 | Sigmoid | 20.29 | 0 | 1 | 0 | 0 | 1 |
| 9 | 0.9 | Sigmoid | 19.90 | 0 | 0 | 0 | 0 | 1 |
| 3 | 0.3 | TanH | 10.31 | 1 | 0 | 1 | 0 | 0 |
| 6 | 0.3 | TanH | 14.47 | 0 | 1 | 1 | 0 | 0 |
| 9 | 0.3 | TanH | 24.19 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0.6 | TanH | 20.85 | 1 | 0 | 0 | 1 | 0 |
| 6 | 0.6 | TanH | 15.40 | 0 | 1 | 0 | 1 | 0 |
| 9 | 0.6 | TanH | 12.49 | 0 | 0 | 0 | 1 | 0 |
| 3 | 0.9 | TanH | 14.72 | 1 | 0 | 0 | 0 | 0 |
| 6 | 0.9 | TanH | 10.53 | 0 | 1 | 0 | 0 | 0 |
| 9 | 0.9 | TanH | 17.38 | 0 | 0 | 0 | 0 | 0 |

Dummy Variables:

x1 = 1 if 3 PE, 0 if not

x2 = 1 if 6 PE, 0 if not

x3 = 1 if LC is .3, 0 if not

x4 = 1 if LC is .6, 0 if not

x5 = 1 if Sigmoid, 0 if not

*Table 28. Linear model for Function 2 ANN results*

Table 29 contains the SPSS output of the results of the regression of the model in Equation 9 for the Function 2 data. The criterion for inclusion in the stepwise regression was a probability of an F-statistic of less than or equal to 0.10. As expected, the variable x5, corresponding to transfer function type, was highly significant. However, what is notable by its overwhelming significance in the ANOVA is the three-way interaction between the factors. The fact that this interaction is more significant than the effect on the model of function type is another strong suggestion that the ANN models are much more robust than the regression models. No individual factor or model parameter appears to dominate.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .860 | .739 | .723 | 4.4824 |
| 2 | .889 | .790 | .762 | 4.1577 |

**ANOVA**[c]

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 911.475 | 1 | 911.47 | 45.37 | .000[a] |
| | Residual | 321.465 | 16 | 20.092 | | |
| | Total | 1232.94 | 17 | | | |
| 2 | Regression | 973.645 | 2 | 486.82 | 28.16 | .000[b] |
| | Residual | 259.295 | 15 | 17.286 | | |
| | Total | 1232.94 | 17 | | | |

a. Predictors: (Constant), X1X4X5

b. Predictors: (Constant), X1X4X5, X5

c. Dependent Variable: Y

*Table 29. SPSS output for Function 2 ANN linear models*

As with the ANN models for the Function 1 data, the linear model was again altered to eliminate the variable x5 (transfer function type) from the main and interaction effects. The ANOVA of this linear regression is in Table 30. After eliminating the results associated with sigmoid-based ANN models, the remaining ANN models show no significant variables at all in the linear model. This may be due to a combination of the low variance of the results and the small number of degrees of freedom for the ANOVA[1]. There may not be enough information to determine the significant interactions.

---

[1] It should be noted, however, that the small degrees of freedom limitation applies to all three functions.

**ANOVA[h]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 31.401 | 1 | 31.401 | 1.586 | .248[a] |
| | Residual | 138.554 | 7 | 19.793 | | |
| | Total | 169.956 | 8 | | | |
| 2 | Regression | 68.678 | 2 | 34.339 | 2.034 | .212[b] |
| | Residual | 101.278 | 6 | 16.880 | | |
| | Total | 169.956 | 8 | | | |
| 3 | Regression | 104.685 | 3 | 34.895 | 2.673 | .158[c] |
| | Residual | 65.271 | 5 | 13.054 | | |
| | Total | 169.956 | 8 | | | |
| 4 | Regression | 126.387 | 4 | 31.597 | 2.901 | .163[d] |
| | Residual | 43.569 | 4 | 10.892 | | |
| | Total | 169.956 | 8 | | | |
| 5 | Regression | 149.256 | 5 | 29.851 | 4.326 | .129[e] |
| | Residual | 20.699 | 3 | 6.900 | | |
| | Total | 169.956 | 8 | | | |
| 6 | Regression | 161.378 | 6 | 26.896 | 6.271 | .144[f] |
| | Residual | 8.577 | 2 | 4.289 | | |
| | Total | 169.956 | 8 | | | |
| 7 | Regression | 167.899 | 7 | 23.986 | 11.665 | .222[g] |
| | Residual | 2.056 | 1 | 2.056 | | |
| | Total | 169.956 | 8 | | | |

[a] Predictors: (Constant), X1X3

[b] Predictors: (Constant), X1X3, X2

[c] Predictors: (Constant), X1X3, X2, X3

[d] Predictors: (Constant), X1X3, X2, X3, X2X4

[e] Predictors: (Constant), X1X3, X2, X3, X2X4, X1X4

[f] Predictors: (Constant), X1X3, X2, X3, X2X4, X1X4, X4

[g] Predictors: (Constant), X1X3, X2, X3, X2X4, X1X4, X4, X1

[h] Dependent Variable: Y

*Table 30. ANOVA of Function 2 ANN linear model eliminating sigmoid models*

On the other hand, eliminating the hyperbolic tangent-based ANN models and performing the regression again shows that the interaction between number of PE and learning coefficient is highly significant (Table 31). This is likely due to the larger variance imparted to the model by the large RMSE value of ANN model 4 (Table 28). The very strong interaction (F = 58.725) between the number of processing elements and the learning coefficient explains over 87 percent of the prediction in the model (adjusted R-squared = 0.878).

Taking all this into consideration, it remains clear that interactions between experimental factors predominate in the results of the ANN models. This may be additional evidence that ANN models are more robust and interconnected than MLR models.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .945[a] | .893 | .878 | 3.5725 |

[a] Predictors: (Constant), X1X4

**ANOVA[b]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 749.494 | 1 | 749.494 | 58.725 | .000[a] |
| | Residual | 89.339 | 7 | 12.763 | | |
| | Total | 838.832 | 8 | | | |

[a] Predictors: (Constant), X1X4

[b] Dependent Variable: Y

*Table 31. SPSS output for Function 2 ANN linear model eliminating hyperbolic tangent models*

| Function Type | Interaction Terms | y | Main Effects | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | x1 | x2 | x3 | x4 | x5 | x6 |
| Linear | 0 | 12.61 | 1 | 0 | 0 | 0 | 1 | 0 |
| Poly-2 | 0 | 13.12 | 0 | 1 | 0 | 0 | 1 | 0 |
| Poly-3 | 0 | 20.00 | 0 | 0 | 1 | 0 | 1 | 0 |
| Exp | 0 | 8.10 | 0 | 0 | 0 | 1 | 1 | 0 |
| Power | 0 | 7.56 | 0 | 0 | 0 | 0 | 1 | 0 |
| Linear | 1 | 12.84 | 1 | 0 | 0 | 0 | 0 | 1 |
| Poly-2 | 1 | 17.17 | 0 | 1 | 0 | 0 | 0 | 1 |
| Poly-3 | 1 | 22.69 | 0 | 0 | 1 | 0 | 0 | 1 |
| Exp | 1 | 11.97 | 0 | 0 | 0 | 1 | 0 | 1 |
| Power | 1 | 16.33 | 0 | 0 | 0 | 0 | 0 | 1 |
| Linear | 2 | 10.89 | 1 | 0 | 0 | 0 | 0 | 0 |
| Poly-2 | 2 | 13.92 | 0 | 1 | 0 | 0 | 0 | 0 |
| Poly-3 | 2 | 27.53 | 0 | 0 | 1 | 0 | 0 | 0 |
| Exp | 2 | 16.14 | 0 | 0 | 0 | 1 | 0 | 0 |
| Power | 2 | 16.91 | 0 | 0 | 0 | 0 | 0 | 0 |

**Dummy Variables:**
x1 = 1 if Linear, 0 if not
x2 = 1 if Poly-2, 0 if not
x3 = 1 if Poly-3, 0 if not
x4 = 1 if Exp, 0 if not
x5 = 1 if 0 interaction terms, 0 if not
x6 = 1 if 1 interaction term, 0 if not

*Table 32. Linear model for Function 2 MLR results*

## Function 2 Robustness Analysis: MLR Results

Table 32 details the linear model for the experimental results from the Function 2 data. The binary dummy variables x1 through x4 correspond to function type, while the variables x5 and x6 correspond to the factor, number of interaction terms. The linear model is identical to Equation 10 where the coefficients of the 14 terms describe the main effects and interaction effects of the two experimental factors.

Table 33 contains the SPSS output of the Function 2 results of the regression of the linear model represented by Equation 10. The criterion for inclusion in the stepwise regression process was a probability of an F-statistic of less than or equal to 0.05. Both models are highly significant (at the 1 percent level) and both contain only main effects for the experimental factors. This again suggests that the MLR models are highly sensitive to the nature of the hypothesized function and therefore, not very robust with respect to either function type or number of interaction terms. The R-squared values reinforce this suggestion. Model 2, containing only main effect terms, explains over 76 percent of the variability of the RMSE results.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .800 | .640 | .612 | 3.3133 |
| 2 | .894 | .800 | .766 | 2.5711 |

**ANOVA[c]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 253.309 | 1 | 253.309 | 23.074 | .000[a] |
| | Residual | 142.717 | 13 | 10.978 | | |
| | Total | 396.026 | 14 | | | |
| 2 | Regression | 316.700 | 2 | 158.350 | 23.954 | .000[b] |
| | Residual | 79.326 | 12 | 6.611 | | |
| | Total | 396.026 | 14 | | | |

[a] Predictors: (Constant), X3

[b] Predictors: (Constant), X3, X5

[c] Dependent Variable: Y

*Table 33. R-Squared values and ANOVA for Function 2 MLR linear model*

## Function 3 Linear Model: ANN Results

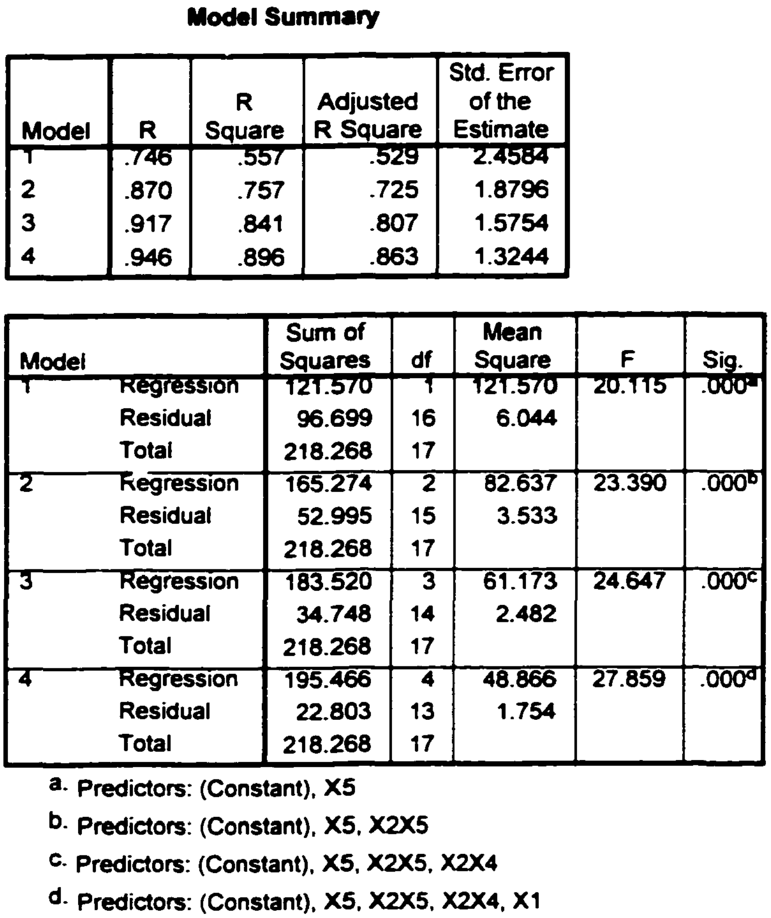
Table 34 details the linear model for the experimental results from the Function 3 data for the 18 ANN models. As in the previous analyses, the dummy variables x1 and x2 correspond to the number of processing elements, x3 and x4 to the level of the learning coefficient, and x5 to transfer function type. The linear model is identical to Equation 9 where the coefficients describe both the main effects and the interactions of the three experimental factors.

|  |  |  |  | Main Effects |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
| PE | LC | TF | y | x1 | x2 | x3 | x4 | x5 |
| 3 | 0.3 | Sigmoid | 12.78 | 1 | 0 | 1 | 0 | 1 |
| 6 | 0.3 | Sigmoid | 16.63 | 0 | 1 | 1 | 0 | 1 |
| 9 | 0.3 | Sigmoid | 12.23 | 0 | 0 | 1 | 0 | 1 |
| 3 | 0.6 | Sigmoid | 12.84 | 1 | 0 | 0 | 1 | 1 |
| 6 | 0.6 | Sigmoid | 13.55 | 0 | 1 | 0 | 1 | 1 |
| 9 | 0.6 | Sigmoid | 10.02 | 0 | 0 | 0 | 1 | 1 |
| 3 | 0.9 | Sigmoid | 12.03 | 1 | 0 | 0 | 0 | 1 |
| 6 | 0.9 | Sigmoid | 19.09 | 0 | 1 | 0 | 0 | 1 |
| 9 | 0.9 | Sigmoid | 10.60 | 0 | 0 | 0 | 0 | 1 |
| 3 | 0.3 | TanH | 10.79 | 1 | 0 | 1 | 0 | 0 |
| 6 | 0.3 | TanH | 7.50 | 0 | 1 | 1 | 0 | 0 |
| 9 | 0.3 | TanH | 7.56 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0.6 | TanH | 10.15 | 1 | 0 | 0 | 1 | 0 |
| 6 | 0.6 | TanH | 5.65 | 0 | 1 | 0 | 1 | 0 |
| 9 | 0.6 | TanH | 5.63 | 0 | 0 | 0 | 1 | 0 |
| 3 | 0.9 | TanH | 8.53 | 1 | 0 | 0 | 0 | 0 |
| 6 | 0.9 | TanH | 6.87 | 0 | 1 | 0 | 0 | 0 |
| 9 | 0.9 | TanH | 10.30 | 0 | 0 | 0 | 0 | 0 |

Dummy Variables:
x1 = 1 if 3 PE, 0 if not x4 = 1 if LC is .6, 0 if not
x2 = 1 if 6 PE, 0 if not x5 = 1 if Sigmoid, 0 if not
x3 = 1 if LC is .3, 0 if not

*Table 34. Linear model for Function 3 ANN results*

Table 35 contains the SPSS output of the results of the regression of the model in equation 9 for the Function 3 data. The criterion for inclusion in the stepwise regression was a probability of an F-statistic of less than or equal to 0.05. As expected, x5 (transfer function type) was again highly significant; however, it was not overwhelmingly so. The variable x1 (number of PE) and two interaction variables were also highly significant.



**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .746 | .557 | .529 | 2.4584 |
| 2 | .870 | .757 | .725 | 1.8796 |
| 3 | .917 | .841 | .807 | 1.5754 |
| 4 | .946 | .896 | .863 | 1.3244 |

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 121.570 | 1 | 121.570 | 20.115 | .000$^a$ |
| | Residual | 96.699 | 16 | 6.044 | | |
| | Total | 218.268 | 17 | | | |
| 2 | Regression | 165.274 | 2 | 82.637 | 23.390 | .000$^b$ |
| | Residual | 52.995 | 15 | 3.533 | | |
| | Total | 218.268 | 17 | | | |
| 3 | Regression | 183.520 | 3 | 61.173 | 24.647 | .000$^c$ |
| | Residual | 34.748 | 14 | 2.482 | | |
| | Total | 218.268 | 17 | | | |
| 4 | Regression | 195.466 | 4 | 48.866 | 27.859 | .000$^d$ |
| | Residual | 22.803 | 13 | 1.754 | | |
| | Total | 218.268 | 17 | | | |

a. Predictors: (Constant), X5

b. Predictors: (Constant), X5, X2X5

c. Predictors: (Constant), X5, X2X5, X2X4

d. Predictors: (Constant), X5, X2X5, X2X4, X1

*Table 35. SPSS output for Function 3 ANN linear models*

By eliminating the factor relating to transfer function type, it is again possible to explore the impact of the remaining model parameters on the performance of the ANN models. The sigmoid-based ANN models were then removed from the linear model, leaving only the hyperbolic tangent models. The results of the ANOVA and the model summary in Table 36 show that, unlike the previous ANN model results, individual factors predominate in this data set. The variable x1, relating to number of processing elements, predominates in the linear model. Interaction terms do not show up in the stepwise regression until the third iteration. Likewise, when the hyperbolic tangent models were removed from the linear model, individual factors predominated. Table 37 shows that variable x2, also corresponding to the number of processing elements, is the first variable to enter the stepwise regression process. This may be an indication that ANN models are not as robust when estimating linear functions as are MLR models. All the models are significant at the 5 percent level.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .656 | .430 | .349 | 1.5807 |
| 2 | .752 | .566 | .421 | 1.4899 |
| 3 | .830 | .689 | .502 | 1.3826 |

**ANOVA[d]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 13.195 | 1 | 13.195 | 5.281 | .055[a] |
| | Residual | 17.491 | 7 | 2.499 | | |
| | Total | 30.686 | 8 | | | |
| 2 | Regression | 17.367 | 2 | 8.683 | 3.912 | .082[b] |
| | Residual | 13.319 | 6 | 2.220 | | |
| | Total | 30.686 | 8 | | | |
| 3 | Regression | 21.128 | 3 | 7.043 | 3.684 | .097[c] |
| | Residual | 9.558 | 5 | 1.912 | | |
| | Total | 30.686 | 8 | | | |

[a] Predictors: (Constant), X1

[b] Predictors: (Constant), X1, X4

[c] Predictors: (Constant), X1, X4, X1X4

[d] Dependent Variable: Y

*Table 36. SPSS output for Function 3 ANN linear model eliminating sigmoid-based models*

An analysis of the signs of the coefficients for the ANN linear models reveals that negative signs are associated predominantly with interaction variables, suggesting that lower RMSE values (better performance) are associated with interactions between factors as opposed to the factors themselves.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .814 | .662 | .614 | 1.7852 |
| 2 | .922 | .850 | .799 | 1.2868 |
| 3 | .953 | .908 | .853 | 1.1028 |
| 4 | .977 | .954 | .908 | .8716 |
| 5 | .990 | .980 | .946 | .6660 |

**ANOVA[f]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 43.704 | 1 | 43.704 | 13.713 | .008[a] |
| | Residual | 22.309 | 7 | 3.187 | | |
| | Total | 66.013 | 8 | | | |
| 2 | Regression | 56.078 | 2 | 28.039 | 16.934 | .003[b] |
| | Residual | 9.935 | 6 | 1.656 | | |
| | Total | 66.013 | 8 | | | |
| 3 | Regression | 59.932 | 3 | 19.977 | 16.425 | .005[c] |
| | Residual | 6.081 | 5 | 1.216 | | |
| | Total | 66.013 | 8 | | | |
| 4 | Regression | 62.974 | 4 | 15.744 | 20.725 | .006[d] |
| | Residual | 3.039 | 4 | .760 | | |
| | Total | 66.013 | 8 | | | |
| 5 | Regression | 64.682 | 5 | 12.936 | 29.165 | .010[e] |
| | Residual | 1.331 | 3 | .444 | | |
| | Total | 66.013 | 8 | | | |

[a] Predictors: (Constant), X2

[b] Predictors: (Constant), X2, X2X4

[c] Predictors: (Constant), X2, X2X4, X1

[d] Predictors: (Constant), X2, X2X4, X1, X2X3

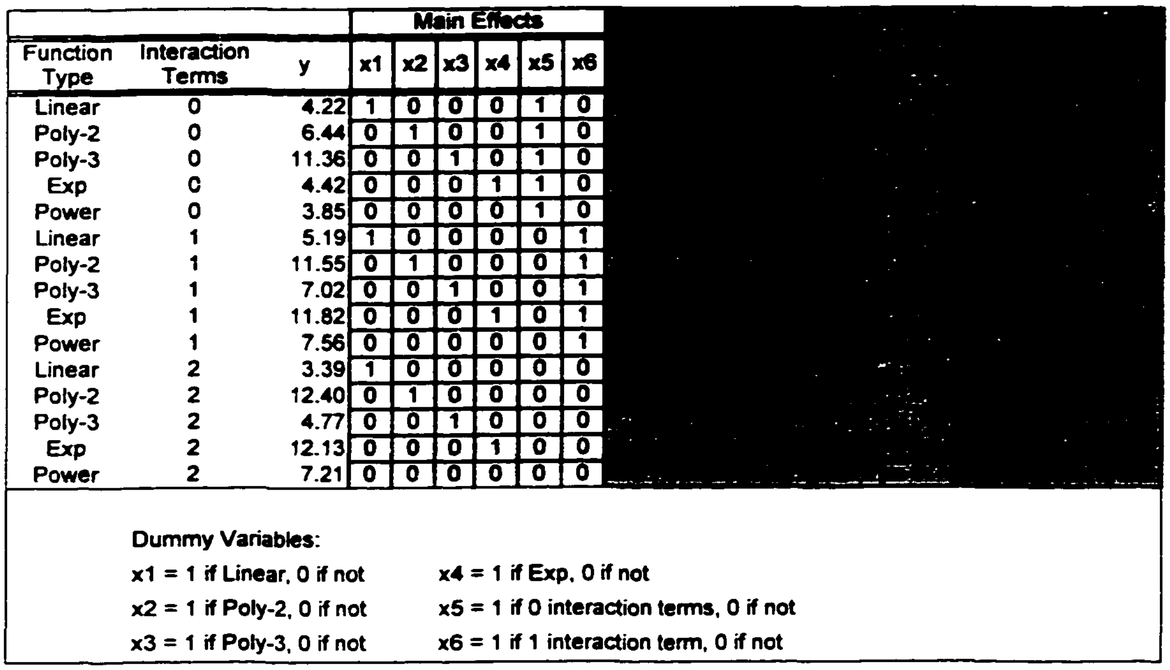
[e] Predictors: (Constant), X2, X2X4, X1, X2X3, X3

[f] Dependent Variable: Y

*Table 37. SPSS output for Function 3 ANN linear model eliminating hyperbolic tangent models*

## Function 3 Linear Model: MLR Results

Table 38 details the linear model for the experimental results from the Function 3 data. The dummy variables x1 through x4 correspond to function type (in this case, a linear function is being estimated) while the variables x5 and x6 correspond to the number of interaction terms. The linear model is based on that in Equation 10, the basic linear model for the analysis of the MLR models for all three data sets.



|  |  |  | Main Effects |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
| Function Type | Interaction Terms | y | x1 | x2 | x3 | x4 | x5 | x6 |
| Linear | 0 | 4.22 | 1 | 0 | 0 | 0 | 1 | 0 |
| Poly-2 | 0 | 6.44 | 0 | 1 | 0 | 0 | 1 | 0 |
| Poly-3 | 0 | 11.36 | 0 | 0 | 1 | 0 | 1 | 0 |
| Exp | 0 | 4.42 | 0 | 0 | 0 | 1 | 1 | 0 |
| Power | 0 | 3.85 | 0 | 0 | 0 | 0 | 1 | 0 |
| Linear | 1 | 5.19 | 1 | 0 | 0 | 0 | 0 | 1 |
| Poly-2 | 1 | 11.55 | 0 | 1 | 0 | 0 | 0 | 1 |
| Poly-3 | 1 | 7.02 | 0 | 0 | 1 | 0 | 0 | 1 |
| Exp | 1 | 11.82 | 0 | 0 | 0 | 1 | 0 | 1 |
| Power | 1 | 7.56 | 0 | 0 | 0 | 0 | 0 | 1 |
| Linear | 2 | 3.39 | 1 | 0 | 0 | 0 | 0 | 0 |
| Poly-2 | 2 | 12.40 | 0 | 1 | 0 | 0 | 0 | 0 |
| Poly-3 | 2 | 4.77 | 0 | 0 | 1 | 0 | 0 | 0 |
| Exp | 2 | 12.13 | 0 | 0 | 0 | 1 | 0 | 0 |
| Power | 2 | 7.21 | 0 | 0 | 0 | 0 | 0 | 0 |

**Dummy Variables:**

x1 = 1 if Linear, 0 if not
x2 = 1 if Poly-2, 0 if not
x3 = 1 if Poly-3, 0 if not
x4 = 1 if Exp, 0 if not
x5 = 1 if 0 interaction terms, 0 if not
x6 = 1 if 1 interaction term, 0 if not

*Table 38. Linear model for Function 3 MLR results*

Table 39 contains the SPSS output of the Function 3 results of the regression of the linear model. The criterion for inclusion in the stepwise regression process was a probability of an F-statistic of less than or equal to 0.183. The four models represented in this table are significant at the 5 and 10 percent levels, but not as significant as those from the Function 1 or 2 data. The variable x1 shows up as the first variable to enter the stepwise regression. This is consistent with good performance of the linear formulations on the linear data-generating function. Additionally, interactions are more prominent in this ANOVA than in previous analyses of variance, appearing in the second model of the stepwise regression process.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .504 | .254 | .196 | 3.0305 |
| 2 | .600 | .361 | .254 | 2.9199 |
| 3 | .721 | .520 | .390 | 2.6413 |
| 4 | .804 | .646 | .504 | 2.3804 |

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 40.593 | 1 | 40.593 | 4.420 | .056[a] |
| | Residual | 119.393 | 13 | 9.184 | | |
| | Total | 159.986 | 14 | | | |
| 2 | Regression | 57.678 | 2 | 28.839 | 3.383 | .068[b] |
| | Residual | 102.308 | 12 | 8.526 | | |
| | Total | 159.986 | 14 | | | |
| 3 | Regression | 83.244 | 3 | 27.748 | 3.977 | .038[c] |
| | Residual | 76.741 | 11 | 6.976 | | |
| | Total | 159.986 | 14 | | | |
| 4 | Regression | 103.325 | 4 | 25.831 | 4.559 | .024[d] |
| | Residual | 56.661 | 10 | 5.666 | | |
| | Total | 159.986 | 14 | | | |

[a] Predictors: (Constant), X1

[b] Predictors: (Constant), X1, X4X5

[c] Predictors: (Constant), X1, X4X5, X4

[d] Predictors: (Constant), X1, X4X5, X4, X2

*Table 39. SPSS output for Function 3 MLR linear model*

The above factors suggest that the MLR models were more robust when estimating a linear function than the non-linear functions represented by Equations 1 and 2. As expected, the linear formulations of the MLR models performed better than the others, however, the polynomial formulations and the exponential formulations were very robust with respect to this linear function, bringing the robustness of the MLR models closer to that of ANN.

**Summary of ANN/MLR Robustness Analysis**

Table 40 summarizes the analysis of the robustness of the 18 ANN models and the 15 MLR models. The models are divided into three categories: ANN models with the transfer function factor included, ANN models with the transfer function factor eliminated, and MLR models. These three categories are further broken down by data set and the Function being estimated. Finally, an X appears in either the "Main Effects" or the "Interaction Effects" column of the table, depending on whether the first model in the stepwise regression included a main effect or interaction effect predictor variable.

| | | Main Effects | Interaction Effects |
|---|---|---|---|
| ANN Models (with transfer function included) | **Function 1** | X | |
| | Function 2 | | X |
| | **Function 3** | X | |
| ANN Models (without transfer function) | **Function 1** | | X |
| | Function 2 | | X |
| | **Function 3** | X | |
| MLR Models | **Function 1** | X | |
| | Function 2 | X | |
| | **Function 3** | X | |

*Table 40. Summary of ANN/MLR ANOVA analysis*

When sorted by model type, it is evident that main effects predominate in the MLR models. Interaction effects were not significant across all three functions for the MLR models. For the linear models in which all 18 ANN models were included, main effects predominated for Functions 1 and 3. The primary reason for this is the overwhelming significance of the model parameter, "transfer function." Those ANN models with sigmoid-based transfer functions had markedly higher variance than the hyperbolic tangent based models. This contributed to the significance of this parameter in the linear models. For the linear models that contained either sigmoid or hyperbolic tangent ANN models, the interaction effects predominated, suggesting these models are less sensitive to parameter changes than the others.

When sorted by function type, main effects predominate with Function 3, the linear data-generating function. Main effects also are important in Function 1 with two of three model types having a main effect model as the initial regression model in the stepwise regression. Interaction effects appear to be significant in the models estimating Function 2. It is notable that, for this non-linear function, the interaction effects were more significant than the effect of transfer function type for all 18 ANN models.

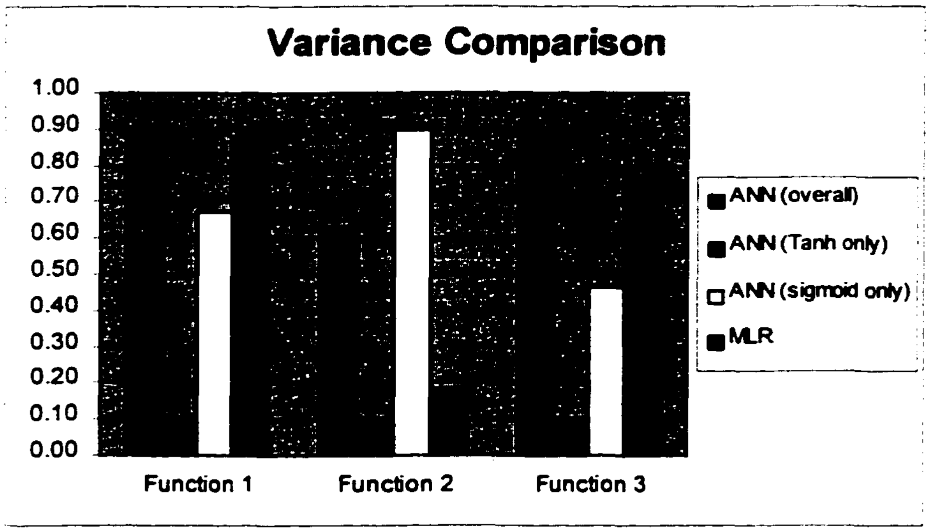


*Figure 3. Variance Comparison between modeling techniques*

Low variance is associated with robust predictive modeling techniques. Figure 3 is a comparison of the variance of the results of the ANN models (including sigmoid and hyperbolic tangent only) and MLR models. The variances are scaled between 0.1 and 0.9 to allow for comparison between functions. The hyperbolic tangent-based ANN models clearly have the lowest variance across all function types. Because of the sigmoid ANN models, the overall ANN model variance is generally higher across all three functions.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .663 | .440 | .397 | 33.8886 |
| 2 | .831 | .691 | .639 | 26.2016 |
| 3 | .940 | .883 | .851 | 16.8181 |
| 4 | .984 | .969 | .956 | 9.1144 |

**ANOVA**[e]

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 11729.639 | 1 | 11729.639 | 10.214 | .007[a] |
| | Residual | 14929.646 | 13 | 1148.434 | | |
| | Total | 26659.285 | 14 | | | |
| 2 | Regression | 18420.994 | 2 | 9210.497 | 13.416 | .001[b] |
| | Residual | 8238.291 | 12 | 686.524 | | |
| | Total | 26659.285 | 14 | | | |
| 3 | Regression | 23547.934 | 3 | 7849.311 | 27.751 | .000[c] |
| | Residual | 3111.352 | 11 | 282.850 | | |
| | Total | 26659.285 | 14 | | | |
| 4 | Regression | 25828.568 | 4 | 6457.142 | 77.730 | .000[d] |
| | Residual | 830.717 | 10 | 83.072 | | |
| | Total | 26659.285 | 14 | | | |

a. Predictors: (Constant), X3

b. Predictors: (Constant), X3, X2

c. Predictors: (Constant), X3, X2, X4

d. Predictors: (Constant), X3, X2, X4, X4X5

e. Dependent Variable: Y

*Table 41. SPSS output for MLR linear model of sample size 50 excursion*

## Model Robustness for Sample Size 50 Excursion

Table 41 contains the SPSS output for the linear model (Equation 10) regressed on the RMSE results of the MLR models using the Function 1 data with the larger sample size. Each of the linear regression models in the ANOVA table is highly significant (at the 1 percent level) and main effects predominate. Main effects account for over 85 percent of the prediction in this linear model.

In Table 42, the model summary and ANOVA are detailed for the linear model (Equation 9) regressed on the RMSE results for the ANN models using the Function 1 data and the larger sample size. Each of the linear models in the ANOVA is very highly significant. As expected, the variable x5, corresponding to transfer function type is overwhelmingly predominant in the linear model, with an adjusted R-squared value of 0.936. A three-way interaction between the factors is also significant in the models.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .970 | .940 | .936 | 9.4548 |
| 2 | .981 | .962 | .957 | 7.7276 |
| 3 | .986 | .972 | .966 | 6.8610 |

**ANOVA**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 22425.955 | 1 | 22425.955 | 250.868 | .000[a] |
| | Residual | 1430.296 | 16 | 89.393 | | |
| | Total | 23856.251 | 17 | | | |
| 2 | Regression | 22960.520 | 2 | 11480.260 | 192.250 | .000[b] |
| | Residual | 895.731 | 15 | 59.715 | | |
| | Total | 23856.251 | 17 | | | |
| 3 | Regression | 23197.226 | 3 | 7732.409 | 164.264 | .000[c] |
| | Residual | 659.024 | 14 | 47.073 | | |
| | Total | 23856.251 | 17 | | | |

[a] Predictors: (Constant), X5

[b] Predictors: (Constant), X5, X1X3X5

[c] Predictors: (Constant), X5, X1X3X5, X3

*Table 42. SPSS output for ANN linear model of excursion (with transfer function)*

Eliminating the sigmoid-based ANN models as well as the variable in the linear model corresponding to transfer function type gives very different results from those obtained from the models trained on samples of size n = 25. Table 43 contains the SPSS output with the ANOVA based solely on ANN models using the hyperbolic tangent function. In this linear model, main effects account for the preponderance of the variability of the results, which is inconsistent with the previous linear model outcomes for the ANN models. Interactions do not appear in the stepwise regression until the fifth iteration. All the models are significant at the 5 percent level.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .692 | .479 | .404 | 6.3453 |
| 2 | .846 | .715 | .620 | 5.0643 |
| 3 | .895 | .802 | .683 | 4.6280 |
| 4 | .944 | .890 | .781 | 3.8503 |
| 5 | .981 | .963 | .901 | 2.5892 |
| 6 | .994 | .988 | .951 | 1.8159 |

**ANOVA**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 258.632 | 1 | 258.632 | 6.424 | .039[a] |
| | Residual | 281.842 | 7 | 40.263 | | |
| | Total | 540.474 | 8 | | | |
| 2 | Regression | 386.592 | 2 | 193.296 | 7.537 | .023[b] |
| | Residual | 153.883 | 6 | 25.647 | | |
| | Total | 540.474 | 8 | | | |
| 3 | Regression | 433.380 | 3 | 144.460 | 6.745 | .033[c] |
| | Residual | 107.094 | 5 | 21.419 | | |
| | Total | 540.474 | 8 | | | |
| 4 | Regression | 481.175 | 4 | 120.294 | 8.114 | .033[d] |
| | Residual | 59.299 | 4 | 14.825 | | |
| | Total | 540.474 | 8 | | | |
| 5 | Regression | 520.362 | 5 | 104.072 | 15.524 | .024[e] |
| | Residual | 20.112 | 3 | 6.704 | | |
| | Total | 540.474 | 8 | | | |
| 6 | Regression | 533.879 | 6 | 88.980 | 26.983 | .036[f] |
| | Residual | 6.595 | 2 | 3.298 | | |
| | Total | 540.474 | 8 | | | |

[a] Predictors: (Constant), X3

[b] Predictors: (Constant), X3, X4

[c] Predictors: (Constant), X3, X4, X1

[d] Predictors: (Constant), X3, X4, X1, X2

[e] Predictors: (Constant), X3, X4, X1, X2, X2X4

[f] Predictors: (Constant), X3, X4, X1, X2, X2X4, X2X3

*Table 43. SPSS output for ANN linear model of excursion (w/0 sigmoid models)*

Eliminating the hyperbolic tangent-based ANN models from the linear model and running the regression generates a result that is similar to the pattern seen with the ANN models for the smaller sample sizes. Interactions between the factors again predominate. Table 44 contains the SPSS output for this linear model. Two-way interactions between the factors are the only significant variables. Main effects are not present. The entering criterion had to be raised to a probability of an F-statistic less than or equal to 0.30 in order to capture the second interaction variable. The F-statistics for both variables are significant at the 5 percent level.

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|---|---|---|---|---|
| 1 | .775 | .601 | .544 | 7.1240 |
| 2 | .826 | .683 | .577 | 6.8596 |

**ANOVA[c]**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 534.564 | 1 | 534.564 | 10.533 | .014[a] |
| | Residual | 355.257 | 7 | 50.751 | | |
| | Total | 889.821 | 8 | | | |
| 2 | Regression | 607.493 | 2 | 303.747 | 6.455 | .032[b] |
| | Residual | 282.328 | 6 | 47.055 | | |
| | Total | 889.821 | 8 | | | |

[a] Predictors: (Constant), X1X3

[b] Predictors: (Constant), X1X3, X2X3

[c] Dependent Variable: Y

*Table 44. SPSS output for ANN linear model of excursion (w/o TanH models*

## Summary of Results

In this section, the significant findings are discussed, to include the significance of the transfer function type, the sensitivity of ANN and MLR models to training sample size, the robustness of ANN and MLR models, and the contributions of interactions among parameters to model performance.

### Significance of transfer function type

For the ANN models in this research, and the type of data being analyzed, the hyperbolic tangent transfer function performed much better than the sigmoid transfer function. The models with hyperbolic tangent functions had lower mean RMSE values as well as lower variances across all three data sets. The ANN models were highly sensitive to changes in transfer function type, masking the significance of factor interactions in the linear models.

That the sensitivity to transfer function type is so high across several different types of data relationships may be an indication that the sigmoid function was an inappropriate transfer function for this type of mapping problem.

The issue of appropriateness of transfer function type for a specific modeling problem is still an area for ongoing research. Caudill and Butler (1992) suggest that the most effective neural networks use a sigmoidal, or S-shaped, transfer function, and that the "...exact form of the sigmoid function is not particularly important; it is merely important that the function be monotonically increasing and bounded with both lower and upper limits" (p. 6). However, it is clear that there is a marked difference between the performance of the sigmoid function (Equation 5) and the hyperbolic tangent function (Equation 6) at least as far as this study is concerned. Both functions are monotonically increasing and have an upper bound of $+1$, while the sigmoid function has a lower bound of 0 and the hyperbolic tangent function a lower bound of $-1$. The hyperbolic tangent function performed significantly better in terms of lower mean and variance for the RMSE model results.

On the other hand, most of the examples from the literature in which the transfer function type was mentioned used the sigmoid function (Equation 5). Markham and Rakes (1998) also adopted the sigmoid function; however, they attempted to optimize their ANN model by manipulating transfer function type as well as number of processing elements and hidden layers. They determined that the sigmoidal transfer function performed better than the hyperbolic tangent function. However, their simulated data was generated using a simple linear function with one independent variable and a normally distributed error term. It is possible that a sigmoid function is more suited to a simple linear data relationship.

This researcher concludes based on the evidence of these experiments, that the hyperbolic tangent function is generally more suitable as an activation function for backpropagation ANN with multiple inputs and one output, and used as predictive models. However, further research should explore, in both a practical and theoretical way, the suitability of various nonlinear activation or transfer functions for backpropagation artificial neural networks. This is addressed again in the following chapter.

### Sensitivity of ANN and MLR models to training sample size

One of the premises under which this study was conducted was that a high signal to noise ratio in the data set contributes to a more accurate predictive model with a lower variance. One way to achieve a high signal to noise ratio is to increase the number of training samples in the data set. Previous research on the effects of sample size on model performance has shown that the performance of both MLR and ANN models improves when a larger training data set is used (Markham and Rakes, 1998; Smith and Mason, 1997; Marquez et al., 1991).

However, it is not always possible to obtain a sufficient number of data points in a modeling problem. Very often, data is sparse and the effects of noise on the quality of the data set is larger. Training sample sizes were kept intentionally small ($n < 50$) in this study to provide a more realistic experimental scenario in which data set sizes might be more reflective of the actual data available.

The experimental results of this study suggest that, without considering robustness, either MLR or ANN modeling approaches work well with small sample sizes. The performance of the best ANN models (hyperbolic tangent) was not statistically different from that of the MLR models. This may have been because the amount of noise imparted to the data through the error term of the data-generating function was insufficiently large relative to the sample size for a detectable difference in performance.

The results of the experiments conducted with the larger sample size of 50 showed a marked improvement in the ANN model performance. There was no improvement in the MLR models with this larger sample size. It can be inferred that ANN models are more sensitive to sample size than MLR models, and that improvement takes place in ANN models at a faster rate with increases in training sample size than the rate of improvement for MLR models with a comparable training sample size increase.

### Robustness of ANN and MLR models

Variance of the RMSE results from model to model when estimating a particular function is a measure of the sensitivity of the model to changes in model formulation. A predictive modeling technique may be considered robust if variations in model formulation do not cause a disproportionately large change in model performance (as measured by a lower-the-better RMSE value).

The hyperbolic tangent-based ANN models appear to be the most robust. The scaled comparison of variances presented in Figure 2 clearly shows that the lowest variances are consistently associated with the hyperbolic tangent ANN models, although there is not a statistically significant difference in the variances of the hyperbolic tangent models and the MLR models for Function 2. This variance was consistently low for the estimates of three widely differing function types, which tends to point to ANN models as being a good first choice for building predictive models in the absence of knowledge about the functional data relationships.

An unexpected finding was the strong and robust performance of the simple linear formulation of the regression function. The linear MLR models (with 0, 1, or 2 interaction terms) actually performed better (in terms of mean RMSE) in estimating Functions 2 (exponential) and 3 (linear) than the best ANN models. This might have been expected for Function 3, but not Function 2. The exponential and power model formulations performed predictably better on the Function 2 data; however, there was no significant difference in estimating performance between the exponential, power, and linear models.

This finding is also consistent with the standard practice in multivariate linear regression modeling of starting the process with a linear formulation, then proceeding to improve the model fit through either polynomial or log transformations of the linear terms (Mendenhall and Sincich, 1995).

### Contribution of interactions to model performance

The ANOVA analysis of the experimental results showed that MLR models were much more sensitive to changes in individual parameters than the ANN models. The model parameter that most often generated the highest variability in the MLR models was the hypothesized function type. This was an expected conclusion, and suggests that if an analyst is unsure about the underlying functional relationship of a data set, or a clear function type does not become evident after several trial and error scatter plots, then it would be safer to build a model using a neural network.

By contrast, ANOVA on the ANN model results shows the overwhelming significance of interaction effects on performance variability. Interactions between the experimental factors are associated with lower variances across the board. It may be concluded from this finding that the parallel and fault-tolerant architecture of ANN models captures the subtle nonlinearities in the data. The large number of free parameters (network weights) in a neural network appear to create sufficient redundancy in the network to reduce its sensitivity to a change in a single model parameter.

These experimental results have shown that both ANN and MLR models can obtain a high degree of accuracy on various types of data. However, ANN models using the hyperbolic tangent transfer function were consistently more robust than MLR models. This characteristic suggests that ANN models might be useful as initial "target" models in a predictive modeling methodology. Subsequent MLR and ANN models could be compared to this target, in an effort to improve and refine the predictive model. In the next chapter, a predictive modeling methodology using both ANN and MLR is proposed. Data sets from two applications from the literature are used to validate the modeling methodology.

# CHAPTER V: PROPOSED PREDICTIVE MODELING METHODOLOGY

In general, ANN models were not overwhelmingly superior to MLR models. One should not conclude, therefore, that one technique is invariably superior to the other. However, these two modeling approaches can be very complementary when combined in a methodology that draws from the advantages and strengths of each. As a result of the findings of this research, a methodology has been developed to provide analysts with a rigorous and practical way to build useful and robust predictive models. It is then applied to two cases taken from the literature involving real-world cost estimating problems.

Ideally, a mathematical function is the preferred form of a model relating independent to dependent variables. Such an equation has two advantages: 1) It is portable, easily understandable, and can be readily incorporated into either spreadsheets or computer source code for further analysis, and 2) the visibility of the functional relationships between the variables provides a level of insight into the nature of the process being modeled. A neural network model, with its "black box" nature, is at a comparative disadvantage to the regression equation.

This research, however, suggests that ANN models have the advantage of being more robust with respect to variations in model formulation. Because of this robust nature, an ANN model might be used initially as a "target" model for an analyst to fix a reasonably achievable target value for coefficient of determination (adjusted R-squared). A recent study concluded through experimentation with artificially generated data that neural network models were very often statistically indistinguishable from the "true model", or the data-generating function (Zeng, 1999). The lower variance of the ANN models increases the likelihood of a good first modeling attempt. Subsequent regression models could be built and compared to the initial target ANN model, continually refining this process until a MLR model is achieved that is, if not better, at least statistically indistinguishable from the ANN model.

As a result of this study, a predictive modeling methodology is proposed and evaluated. The following ten-step methodology incorporates both regression and neural network modeling techniques, capitalizing on the strengths of each. It will provide practitioners with a rigorous and structured way to derive the best possible predictive model:

## ANN/MLR Modeling Methodology

1) Step 1: Build a neural network using the independent variables as the input layer, the dependent variable as the output layer, and one hidden layer. The number of processing elements in the hidden layer should be determined by heuristic. Use the hyperbolic tangent transfer function and a learning constant around 0.5 initially.
2) Step 2: Train the neural network using the entire data set as a training set and save the network weights.
3) Step 3: Run the data set through the network with the learning turned off and compare the desired output (y) with the actual result from the network. Calculate the adjusted R-squared value.
4) Step 4: Repeat step 1 through step 3 two more times to build two more networks. With each subsequent network, vary the learning constant slightly up or down.
5) Step 5: Choose the network with the largest R-Squared value as the target model.

6) Step 6: Construct a stepwise linear regression model starting with all the independent variables and no transformed variables. This becomes the baseline regression model. Calculate its R-squared value. If it is larger than the best NN model value, use the linear model.

7) Step 7: If the R-squared is lower than that of the best NN model, compare the output of the linear model against that of the best NN model using a pairwise t-test. If there is a statistical difference in the means of the two results, then it is likely the best model is the ANN model. If there is no statistical difference between the two outputs, it is possible that a better MLR model can be constructed using non-linear transformations of the independent variables. In either case, proceed to step 8.

8) Step 8: Build a scatterplot for each of the independent variables with the independent variable on the X axis and the dependent variable on the Y axis. Add a trendline to this scatterplot using the data analysis functions of the spreadsheet software. Determine the equation for this line and the R-squared value. Go through each of the possible variations of the trendline (logarithmic, exponential, polynomial, etc.), observing the change in the R-squared value. If the R-squared improves, note the nature of the nonlinear relationship to the dependent variable. For example, if the best R-squared is associated with a cubic polynomial relationship, then in the MLR model, additional nonlinear terms should be added to the model reflecting the cubic relationship.

9) Step 9: Reconstruct a more detailed MLR model using the nonlinear transformations of the independent variables that were determined in Step 8.

Perform both a stepwise regression and one in which all the terms are entered in the model. Calculate the predicted output as well as the R-squared.

10) Step 10: Compare the transformed MLR model with both the baseline linear model and the ANN model using both R-squared and a pairwise t-test. If the R-squared of the transformed MLR model is better than the ANN model, use the MLR model. If the R-squared value of the transformed MLR model is still lower than the ANN model, but there is no significant difference between the output of the two models, then the transformed MLR model should still be used. If there is still a statistical difference between both the baseline and the transformed MLR models and the ANN model, the ANN model should be used.

The objective is to use a regression model whenever possible, using the best ANN model as a gauge to validate the effectiveness of the MLR model. The more data available to build the ANN and MLR models, the better this technique should perform.

## An Example Using the Data from de la Garza and Rouhana (1995)

De la Garza and Rouhana (1995) used three different characteristics of carbon steel pipe to build a predictive cost model. The data for their study are shown in Table 45. They compared the traditional linear regression-based parametric model with a neural network model, concluding that the neural network model outperformed the regression models. Using the above modeling methodology, it is shown that de la Garza and Rouhana arrived at their conclusions prematurely; without a thorough analysis of the data.

| Job | X1 Diameter (in) | X2 Number of Elbows | X3 Flange Rating | Y Nominal Cost per 100 ft |
|---|---|---|---|---|
| 1 | 20 | 14 | 250 | 46.1 |
| 2 | 20 | 14 | 150 | [illegible] |
| 3 | 20 | 14 | 100 | 42.1 |
| 4 | [illegible] | [illegible] | [illegible] | [illegible] |
| 5 | 12 | 12 | 100 | 16.8 |
| 6 | [illegible] | [illegible] | [illegible] | [illegible] |
| 7 | 16 | 12 | 100 | 26.3 |
| 8 | 16 | [illegible] | 200 | [illegible] |
| 9 | 4 | 4 | 300 | 2.5 |
| 10 | 24 | 12 | 100 | [illegible] |
| 11 | 16 | 12 | 200 | 28.4 |
| 12 | 20 | 12 | 250 | [illegible] |
| 13 | 6 | 12 | 150 | 6.5 |
| 14 | 24 | [illegible] | 300 | [illegible] |
| 15 | 12 | 4 | 300 | 10.8 |
| 16 | 20 | [illegible] | 100 | [illegible] |

*Table 45. Carbon steel pipe data*

**Step 1.** A neural network was constructed with an input layer of 3 processing elements, corresponding to the 3 independent variables and an output layer of one processing element for the dependent variable. Using the heuristic of Flitman (1997), the number of neurodes in the hidden layer is determined using the following formula:

*Number of hidden neurons = ½ (Inputs + Outputs) + Sqrt(# of training patterns)*

With three inputs, one output, and 16 training patterns, the number of hidden neurodes for the network is set at six. The hyperbolic tangent is used as the transfer function and the learning constant is set at 0.5.

<u>Step 2.</u> The entire data set was used to train the neural network. Normally only a portion of the available data would be used to train a neural network. The remaining exemplars would be withheld as a testing/validation set to determine how well the neural network was able to generalize its learning. However, in this methodology, the entire set was used both to train and evaluate the network so that a residual analysis could be performed and an adjusted R-squared determined, similar to the procedure used in a regression analysis.

<u>Step 3.</u> The weights of the trained network (Network 1) were saved and the backpropagation learning was turned off. The independent variable exemplars were run through the model to generate an estimated y value. This estimate was compared to the desired y values (cost) for each exemplar to calculate an adjusted R-squared for the model. Table 46 contains the desired and actual output, adjusted R-squared, and learning constant for the three networks constructed to determine the "target" model. The adjusted R-squared takes into consideration both the sample size and the number of independent variables in the model. It is considered a more conservative measure of model adequacy than the R-squared (Mendenhall and Sincich, 1995). The adjusted R-squared is given by:

$$R_a^2 = 1 - \frac{n-1}{n-(k+1)}\left(1 - R^2\right), \qquad (11)$$

| Desired | Network 1<br>LC = 0.5 | Network 2<br>LC = 0.7 | Network 3<br>LC = 0.3 |
|---|---|---|---|
| 46.10 | 44.00 | 44.06 | 43.68 |
| 43.20 | 44.19 | 43.45 | 43.07 |
| 42.10 | 43.81 | 41.88 | 41.99 |
| 1.90 | 5.79 | 5.38 | 5.12 |
| 16.80 | 15.12 | 14.94 | 14.70 |
| 11.70 | 12.44 | 12.90 | 12.84 |
| 26.30 | 27.25 | 25.90 | 25.87 |
| 26.10 | 25.00 | 24.27 | 24.66 |
| 2.50 | 5.04 | 4.50 | 3.73 |
| 50.20 | 46.41 | 45.90 | 45.68 |
| 28.40 | 28.38 | 27.10 | 28.01 |
| 41.30 | 41.07 | 41.68 | 41.35 |
| 6.50 | 7.24 | 6.94 | 6.65 |
| 42.30 | 41.65 | 41.28 | 42.32 |
| 10.80 | 8.68 | 7.01 | 5.97 |
| 28.90 | 30.12 | 30.07 | 29.99 |
| Adj R-squared | 0.9852 | 0.9834 | 0.9812 |

*Table 46. Performance of ANN models on pipe data*

where *n* is the sample size and *k* is the number of independent variables. The R-squared is calculated by taking the square of the coefficient of correlation between the desired and actual output.

Step 4. Two more ANN models (Network 2 and Network 3) were constructed and trained using the same data (Table 46). The learning constant was varied by 0.2 from Network 1 in each direction for these two networks.

Step 5. Although the adjusted R-squared values for the three ANN models were very close, Network 1 had the highest value and was chosen as the target model.

Step 6. SPSS was used to construct a baseline linear regression model. A stepwise regression procedure resulted in the following linear model:

$$y = -17.926 + 2.205x_1 + 1.012x_2, \tag{12}$$

with an adjusted R-squared of 0.94. Since this value is lower than the ANN models, we must proceed to step 7.

**Step 7.** Although the R-squared value of equation 9 is less than that of Network 1, a paired t-test comparing the output of Network 1 with the output of the model in equation 9 indicates there is insufficient evidence to reject the hypothesis that they are drawn from the same population (probability that T <= t-critical is 0.9428). However, it is possible that a better MLR model can be constructed using non-linear transformations of the independent variables.

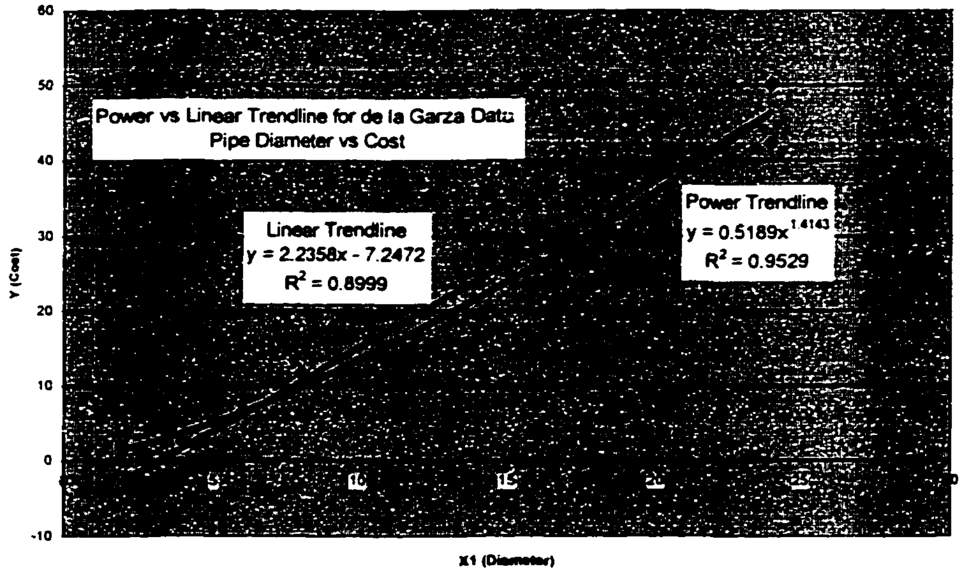


*Figure 4. Scatterplot and trendlines for X1 vs Y*

**Step 8.** Figures 4 through 6 show two-way scatterplots of each of the three independent variables against the dependent cost variable. A baseline linear trendline was calculated for each scatterplot along with the associated R-squared. Then a sequence of non-linear trendlines was fitted to the data in each of the scatterplots. As can be seen in figures 5 and 6, as well as the R-squared values in table 47, there is very little correlation between the variables X2 and X3 and Y. The scatterplot analysis revealed that a power function (model coefficients in the exponents) provides a much better fit for the data in figure 4. Therefore a power model is constructed in the next step.

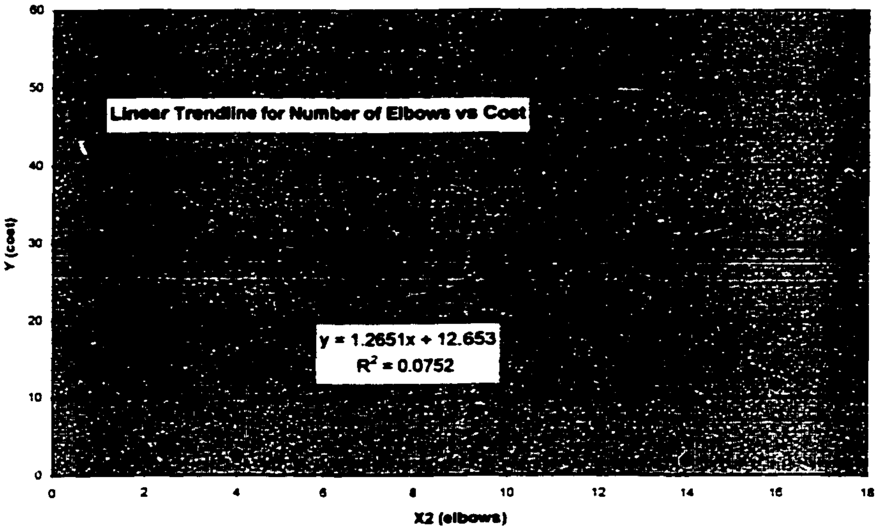


*Figure 5. Scatterplot and trendline for X2 vs Y*

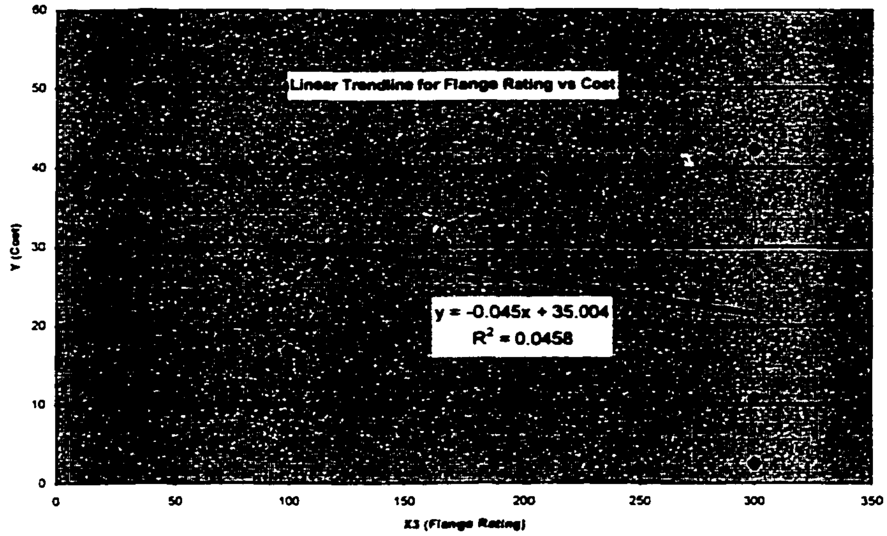


*Figure 6. Scatterplot and trendline for X3 vs Y*

<u>Step 9.</u> Another regression model was constructed using power transformations of the linear terms in the baseline model. In order to perform the stepwise regression, the

| | Linear | Poly-2 | Poly-3 | Log | Exp | Power |
|---|---|---|---|---|---|---|
| X1 vs Y | 0.8999 | 0.9121 | 0.9189 | 0.7557 | 0.912 | 0.9629 |
| X2 vs Y | 0.0752 | 0.244 | [illegible] | 0.1213 | 0.0551 | 0.1034 |
| X3 vs Y | 0.0458 | 0.046 | [illegible] | 0.049 | 0.1161 | 0.1158 |

*Table 47. R-squared values for partial regression plots*

equation must be in a linear form. Taking the natural logarithm of both sides of the power function makes this transformation possible. The resulting model,

$$y = 0.07x_1^{1.399}x_2^{0.569}x_3^{0.139}, \tag{13}$$

has an adjusted R-squared value of 0.997, a considerable improvement over both the baseline MLR model and the Network 1 ANN model.

**Step 10.** Table 48 summarizes the comparison between the Network 1 ANN model, the baseline MLR model, and the transformed power MLR model. There is no statistically significant difference between any of these three models; however, the power MLR model has a larger adjusted R-squared, implying it does a better job of explaining the variability in the cost data. It is also interesting to note that the ANN model has the lowest variance of the three models.

| | Power | Network 1 | Baseline |
|---|---|---|---|
| R-Squared (adj) | 0.997 | 0.985 | 0.940 |
| Variance | 248.088 | 245.404 | 258.137 |

*Table 48. Comparison of ANN and MLR models*

The steel pipe cost data from de la Garza and Rouhana (1995) submitted readily to linear regression analysis, providing an unusually well-fitted model after several attempts at non-linear transformations of the variables. However, unless a thorough parametric modeling process is followed, an analyst may easily reach the premature conclusion that a neural network model is generally better than a regression model. This was the case in de la Garza and Rouhana (1995).

Example 2: Data from Creese and Li (1995)

Creese and Li (1995) also compared neural network cost models to parametric regression cost models using cost data on 12 bridges (Table 49). The Creese and Li (1995) data set is similar to de la Garza and Rouhana (1995) in that both have a small number of exemplars (12 and 16 respectively) as well as three independent variables or cost drivers.

| | Web Vol (ft$^3$) | Deck Vol (ft$^3$) | Steel Wt (lb) | Actual Cost ($) |
|---|---|---|---|---|
| Bridge | X1 | X2 | X3 | Y |
| 1 | 662.86 | 542.34 | 527.98 | 74,982 |
| 2 | 791.15 | 566.72 | 651.08 | 87,602 |
| 3 | 265.58 | 254.54 | 352.67 | 45,400 |
| 4 | 781.41 | 737.70 | 676.12 | 92,850 |
| 5 | 336.88 | 753.38 | 434.06 | 75,000 |
| 6 | 348.05 | 830.25 | 394.41 | 60,894 |
| 7 | 455.18 | 567.50 | 535.27 | 61,354 |
| 8 | 1164.17 | 892.97 | 834.72 | 79,512 |
| 9 | 1661.65 | 2825.00 | 1316.25 | 201,600 |
| 10 | 1665.04 | 2484.38 | 1168.81 | 194,599 |
| 11 | 383.90 | 408.30 | 367.00 | 55,113 |
| 12 | 2320.00 | 1444.00 | 1331.00 | 174,000 |

*Table 49. Bridge cost data*

Creese and Li (1995) concluded that ANN models outperformed MLR models using R-squared as a performance criterion. However, they used only simple linear formulations of the independent variables for the regression equation, never attempting to fit the data to a nonlinear transformation of the independent variables.

Using the above ten-step methodology, the most appropriate linear model was based on a cubic transformation of the independent variables. Such a regression model performed slightly better than neural network models constructed using the Flitman (1997) heuristic and a hyperbolic tangent transfer function although not quite as well as the neural model constructed by Creese and Li (1995). Table 50 compares the results of Creese and Li (1995) and the methodology in this research. As with the models in de la Garza and Rouhana, there is not a statistically significant difference between any of the models in Table 50 (at the 5-percent significance level). However, the probability that the means of the cubic model results and the 10-step network results (based on a paired t-test) are the same is only 0.118, suggesting that the cubic MLR model is fairly close to being significantly better.

| | Linear Model | Cubic Model | Creese/Li Network | "10-step" Network |
|---|---|---|---|---|
| R-squared | 0.970 | 0.989 | 0.991 | 0.971 |
| R-squared (adj) | 0.958 | 0.985 | 0.988 | 0.960 |

*Table 50. Creese and Li vs 10-step methodology*

## Summary

In this chapter, a predictive modeling methodology was proposed that combines the use of ANN and MLR models. The robust nature of ANN models makes them good candidates for an initial target model. The ultimate form of the predictive model may be either an MLR equation or an ANN; however, by using both modeling techniques, the methodology can increase the level of confidence in the accuracy and robustness of the model.

Applying the methodology to the two case studies from the literature confirms that a combined approach can result in a better model than one or the other technique alone. The example from de-la-Garza and Rouhana (1995) confirmed the utility of the ANN model, but also pointed out the incomplete regression analysis. In the Creese and Li (1995) example, although the ANN is the better model (using R-squared), it is shown that a cubic MLR model may be close enough to be the more useful of the two.

# CHAPTER VI: CONCLUSIONS AND FURTHER RESEARCH

In this chapter, the conclusions of this research are summarized, the limitations of the research are noted, and the contribution to the literature is described. In addition, areas for further research are discussed.

## Summary of Conclusions

Hyperbolic tangent-based ANN models can serve as credible and effective surrogates for least squares regression models. They are accurate and robust with respect to changes in network topology. However, the ANN models in this research were not overwhelmingly superior to the MLR models. One should not conclude, therefore, that one technique is invariably superior to the other.

As the data available for training increases, the signal to noise ratio also increases and ANN model performance appears to improve at a faster rate than that of MLR models in response to the same expanded data set.

Linear formulations of MLR models exhibit surprisingly robust characteristics even when estimating non-linear functions. This is testimony to the power and utility of the least squares estimator.

If the training sample size is less than 50, hyperbolic tangent neural network models may not necessarily produce better results than regression models in terms of lower RMSE or higher R-squared. However, because of their lower variance, they could be used in conjunction with MLR models to provide a more complete modeling methodology. Based on the experimental results and conclusions, a predictive modeling methodology has been developed that capitalizes on the advantages of both neural network and regression approaches and may assist practitioners in constructing accurate and robust predictive models. Applying the methodology to two case studies from the application literature showed that this approach can result in a better model than one or the other technique alone.

## Limitations of Research

The results of this research are limited by the type of data, the formulations of the ANN and MLR models used in the experiments, the sample sizes chosen, and the size of the input vector.

The research relied on simulated data with artificially generated noise in the form of a normally distributed error term. The functions used to generate the data pools were chosen because they represented widely varying types of data relationships; however, it is not implied that the three data generating functions are representative of all the potential data types a practitioner might be faced with in a predictive modeling situation. Additionally, the ranges of the independent variables in the data-generating functions may have affected the comparative performance.

The ANN and MLR model formulations used were designed to be indicative of “real world” approaches an analyst might use in dealing with various data sets. This research is, therefore, limited to a fairly narrow range of ANN topologies. Other combinations of activation function, learning constant, momentum, number of processing elements, and training algorithm could have been used in structuring the ANN models.

As was discussed in the research methodology chapter, the sample size was fixed at $n = 25$. The researcher does not feel this is a significant limitation of the research, as it has been shown that performance of both ANN and MLR predictive models improves with larger sample sizes.

Finally, the input vector was constrained to four input variables. This limits the generalizability of this research to similar types of regression problems. In actual applications, however, this may not be a practical limitation, as larger input vectors are often "pruned" through techniques such as Principal Components Analysis and stepwise regression to reflect only those independent variables most highly correlated with the dependent variable.

## Contributions

This research provides a theoretical and practical contribution to the predictive modeling literature by quantifying the effect of model formulation on the comparative performance of ANN and MLR, and by providing a predictive modeling methodology based on the combined use of ANN and MLR modeling techniques.

Additionally, linear models of the experimental results were generated that provided insight into the variance contributions of individual model parameters. This extensive ANOVA approach is unique to the study of ANN and MLR, and is also a contribution.

## Further Research

This research attempted to address specific questions regarding the comparative performance of ANN and MLR models. In the process, more questions were raised which might form the basis for further inquiry into this research area. Three areas are discussed in this chapter: 1) Appropriateness of neural network transfer function type for specific modeling problems, 2) Relative rates of performance improvement between ANN and MLR models with increases in sample size (signal-to-noise ratio), 3) Robustness of linear and various nonlinear regression model formulations with respect to varying types of data.

## Transfer Function Type

Caudill and Butler (1992) were quoted in the previous chapter as stating that the "...exact form of the sigmoid function is not particularly important."[2] However, the results of this research suggest otherwise. It is clear that transfer function type has a significant effect on the performance of neural network models used as surrogates for regression models. This research concluded that, because of the consistent and significantly better performance of the hyperbolic tangent function over the sigmoid function, the hyperbolic tangent activation function may be more appropriate in predictive modeling problems in which there is one dependent variable.

Further research into the use of ANN as surrogates to MLR models should include experimentation with various transfer function types. It is still unclear how the transfer function affects the performance of a neural network. It would be useful to know whether the type of neural network problem (regression, classification, etc.), or the type

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[2] By "sigmoid function," Caudill and Butler (1992) are referring to any S-shaped function having the properties of mapping the function argument onto a point between a narrowly defined upper and lower bound, such as 0 and 1, or −1 and +1. In this research, the term "sigmoid function" refers to the logistic function shown in equation 5.

of data relationship (linear, nonlinear) has any bearing on the appropriateness of a certain transfer function type.

A designed experiment could be conducted in which the only manipulated variable would be transfer function type. All other variables such as sample size, input vector, number of processing elements, learning coefficient, and any other model parameters could be held constant to isolate just the effects on performance due to change in transfer function type. In such an experiment, it would be important to test the performance of each of the ANN models on various data sets generated using a variety of linear and nonlinear functions.

A likely outcome of this experiment would be confirmation that the hyperbolic tangent transfer function performs significantly better than other transfer functions for a range of data relationships in neural network models used as surrogates for linear regression models.

## Sensitivity of ANN and MLR Models to Sample Size Increases

Although much experimentation has been done on the effects of sample size on the performance of neural network and regression models, additional experimentation could be done to detect the rate of change of performance of these models given various sample sizes. The objective of such an experiment might be to discover the "inflection points" of the curve describing model performance over sample size. Figure 7 illustrates the hypothetical comparative performance between ANN and MLR models on a given data set. Development of such a series of curves might help define what constitutes "small" and "large" sample sizes for given modeling situations.

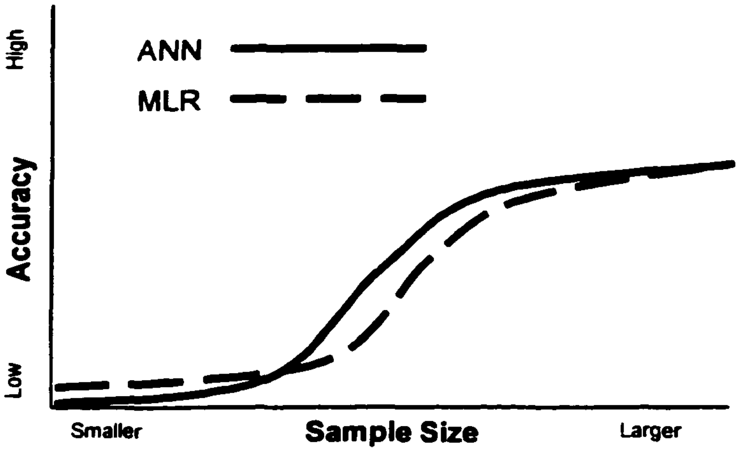

*Figure 7. Rate of change in performance of ANN and MLR vs sample size*

Perhaps more specifically, signal-to-noise (S/N) ratio could be compared against model performance. The S/N ratio takes into consideration the effect of noise, or randomness, in the data. A given sample size can have a variety S/N ratios depending on the quality of the data. Therefore, S/N ratio might be a more effective measure of performance.

## Robustness of Linear MLR Formulations

One of the conclusions of this research was the unexpectedly strong and robust performance of simple linear formulations of the regression function. Further research in the area of predictive modeling techniques should compare the relative robustness of these linear formulations against that of nonlinear (polynomial and log-transformed) formulations. Such an investigation might yield useful information about the utility of simple model formulations for rapid but accurate statistical modeling.

## Concluding Comments

This research has shown that the chief advantage of ANN predictive models over MLR models is their relative insensitivity to changes in model parameters. It has also shown that, within the limitations and scope of the research problem, ANN and MLR predictive models have comparable levels of accuracy. Given these conclusions, this researcher suggests a predictive modeling approach that involves both ANN and MLR models. Such an approach may assist practitioners in constructing accurate and robust predictive models by capitalizing on the advantages of each individual technique.

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# APPENDIX A: NEURAL NETWORK EXPERIMENT MATRIX

| Model | Number of Processing Elements | Learning Coefficient | Transfer Function Type |
|---|---|---|---|
| 1 | 3 | 0.3 | Sigmoid |
| 2 | 6 | 0.3 | Sigmoid |
| 3 | 9 | 0.3 | Sigmoid |
| 4 | 3 | 0.6 | Sigmoid |
| 5 | 6 | 0.6 | Sigmoid |
| 6 | 9 | 0.6 | Sigmoid |
| 7 | 3 | 0.9 | Sigmoid |
| 8 | 6 | 0.9 | Sigmoid |
| 9 | 9 | 0.9 | Sigmoid |
| 10 | 3 | 0.3 | Hyperbolic Tangent |
| 11 | 6 | 0.3 | Hyperbolic Tangent |
| 12 | 9 | 0.3 | Hyperbolic Tangent |
| 13 | 3 | 0.6 | Hyperbolic Tangent |
| 14 | 6 | 0.6 | Hyperbolic Tangent |
| 15 | 9 | 0.6 | Hyperbolic Tangent |
| 16 | 3 | 0.9 | Hyperbolic Tangent |
| 17 | 6 | 0.9 | Hyperbolic Tangent |
| 18 | 9 | 0.9 | Hyperbolic Tangent |

# APPENDIX B: MLR EXPERIMENT MATRIX

| Model | Function Type | Equation | Log-Transformed Equations |
|---|---|---|---|
| 1 | Linear | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$ | N/A |
| 2 | 2nd order polynomial | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 + \beta_3 x_3 + \beta_4 x_4$ | N/A |
| 3 | 3rd order polynomial | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_2^2 + \beta_3 x_3^2 + \beta_4 x_4$ | N/A |
| 4 | Exponential | $y = \frac{\beta_0 x_1^{0.5\beta_1} e^{\beta_2 x_2} \beta_3 x_3}{\beta_4 x_4}$ * | $\ln y = \ln \beta_0 + 0.5\beta_1 \ln x_1 + \beta_2 x_2 + \beta_3 \ln x_3 - \beta_4 \ln x_4$ |
| 5 | Power | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta_3} x_4^{\beta_4}$ * | $\ln y = \ln \beta_0 + \beta_1 \ln x_1 + \beta_2 \ln x_2 + \beta_3 \ln x_3 + \beta_4 \ln x_4$ |
| 6 | Linear with one interaction term | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_3 + \beta_4 x_4$ | N/A |
| 7 | 2nd order polynomial with one interaction term | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 x_3 + \beta_3 x_3 + \beta_4 x_4$ | N/A |
| 8 | 3rd order polynomial with one interaction term | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 + \beta_4 x_4$ | N/A |
| 9 | Exponential with one interaction term | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3} e^{\beta_4 x_4}$ * | $\ln y = \ln \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_3 + \beta_4$ |
| 10 | Power with one interaction term | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3} x_4^{\beta_4}$ * | $\ln y = \ln \beta_0 + \beta_1 \ln x_1 + \beta_2 x_3 \ln x_2 + \beta_3 \ln x_3 + \beta_4 \ln x_4$ |
| 11 | Linear with two interaction terms | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_2 x_3 + \beta_4 x_3 + \beta_5 x_4$ | N/A |
| 12 | 2nd order polynomial with two interaction terms | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 x_3 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | N/A |
| 13 | 3rd order polynomial with two interaction terms | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | N/A |
| 14 | Exponential with two interaction terms | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3 x_4} e^{\beta_4 x_3} e^{\beta_5 x_4}$ * | $\ln y = \ln \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_3 x_4 + \beta_4 x_3 + \beta_5 x_4$ |
| 15 | Power with two interaction terms | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3 x_4} x_4^{\beta_4}$ * | $\ln y = \ln \beta_0 + \beta_1 \ln x_1 + \beta_2 x_3 \ln x_2 + \beta_3 x_4 \ln x_3 + \beta_4 \ln x_4$ |

**** Note: Highlighted rows represent Best case regression model (same specification as true function)**

# APPENDIX C: MLR MODELS

| Function 1 | | | |
|---|---|---|---|
| **Model Number** | **Formulation** | **Stepwise Regression Model** | **Adjusted R-squared** |
| 1a | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$ | $y = -115.815 + 88.367 x_1$ | 0.884 |
| 1b | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$ | $y = -108.619 + 95.925 x_1$ | 0.916 |
| 2a | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 + \beta_3 x_3 + \beta_4 x_4$ | $y = 49.031 + 9.327 x_1^2$ | 0.975 |
| 2b | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 + \beta_3 x_3 + \beta_4 x_4$ | $y = 26.49 + 9.739 x_1^2$ | 0.961 |
| 3a | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_2^2 + \beta_3 x_3^2 + \beta_4 x_4$ | $y = -2.524 + 1.016 x_1^3 + 0.886 x_3^2 + 23.936 x_4$ | 0.999 |
| 3b | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_2^2 + \beta_3 x_3^2 + \beta_4 x_4$ | $y = 96.807 + 1.028 x_1^3 + 0.78 x_3^2$ | 0.997 |
| 4a | $y = \dfrac{\beta_0 x_1^{0.5\beta_1} e^{\beta_2 x_2} \beta_3 x_3}{\beta_4 x_4}$ | $\ln y = 4.199 + 1.055 \ln x_1$ | 0.867 |
| 4b | $y = \dfrac{\beta_0 x_1^{0.5\beta_1} e^{\beta_2 x_2} \beta_3 x_3}{\beta_4 x_4}$ | $\ln y = 3.803 + 1.213 \ln x_1$ | 0.865 |
| 5a | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta_3} x_4^{\beta_4}$ | $\ln y = -0.174 + 0.879 \ln x_1 + 3.245 \ln x_4$ | 0.781 |
| 5b | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta_3} x_4^{\beta_4}$ | $\ln y = 3.559 + 1.341 \ln x_1$ | 0.872 |
| 6a | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_3 + \beta_4 x_4$ | $y = -251.445 + 114.478 x_1$ | 0.901 |
| 6b | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_3 + \beta_4 x_4$ | $y = -204.775 + 111.066 x_1$ | 0.859 |
| 7a | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 x_3 + \beta_3 x_3 + \beta_4 x_4$ | $y = 42.324 + 9.918 x_1^2$ | 0.976 |
| 7b | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 x_3 + \beta_3 x_3 + \beta_4 x_4$ | $y = 32.592 + 9.727 x_1^2$ | 0.981 |

| Model Number | Formulation | Stepwise Regression Model | Adjusted R-squared |
|---|---|---|---|
| 8a | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 + \beta_4 x_4$ | $y = 93.626 + 1.007x_1^3 + 0.875x_3^2$ | 0.997 |
| 8b | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 + \beta_4 x_4$ | $y = 93.265 + 1.007x_1^3 + 0.912x_3^2$ | 0.998 |
| 9a | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3} e^{\beta_4 x_4}$ | $\ln y = 3.951 + 0.289x_1 + 0.03017x_3$ | 0.980 |
| 9b | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3} e^{\beta_4 x_4}$ | $\ln y = 4.008 + 0.288x_1 + 0.043x_3$ | 0.995 |
| 10a | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3} x_4^{\beta_4}$ | $\ln y = 2.931 + 1.416 \ln x_1 + 0.347 \ln x_3$ | 0.926 |
| 10b | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3} x_4^{\beta_4}$ | $\ln y = 3.937 + 1.14 \ln x_1$ | 0.869 |
| 11a | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_2 x_3 + \beta_4 x_3 + \beta_5 x_4$ | $y = -126.717 + 95.407x_1$ | 0.869 |
| 11b | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_2 x_3 + \beta_4 x_3 + \beta_5 x_4$ | $y = -186.923 + 104.153x_1$ | 0.852 |
| 12a | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 x_3 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | $y = 0.797 + 9.771x_1^2 + 0.254x_3^2 x_4$ | 0.977 |
| 12b | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 x_3 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | $y = 29.33 + 9.999x_1^2$ | 0.984 |
| 13a | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | $y = 88.132 + 1.037x_1^3 + 0.25x_3^2 x_4$ | 0.998 |
| 13b | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | $y = 92.794 + 1.023x_1^3 + 0.25x_3^2 x_4$ | 0.999 |
| 14a | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3 x_4} e^{\beta_4 x_3} e^{\beta_5 x_4}$ | $\ln y = 3.927 + 0.281x_1 + 0.04975x_3$ | 0.986 |
| 14b | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3 x_4} e^{\beta_4 x_3} e^{\beta_5 x_4}$ | $\ln y = 3.862 + 0.297x_1 + 0.009799x_3 x_4$ | 0.995 |
| 15a | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3 x_4} x_4^{\beta_4}$ | $\ln y = 3.943 + 1.135 \ln x_1$ | 0.793 |
| 15b | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3 x_4} x_4^{\beta_4}$ | $\ln y = 4.175 + 1.028 \ln x_1$ | 0.814 |

## Function 2

| Model Number | Formulation | Stepwise Regression Model | Adjusted R-squared |
|---|---|---|---|
| 1a | $y=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4$ | $y=-149.92+4.758x_1+43.558x_2+10.226x_3$ | 0.876 |
| 1b | $y=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4$ | $y=-214.794+6.106x_1+64.393x_2+10.571x_3$ | 0.819 |
| 2a | $y=\beta_0+\beta_1x_1^2+\beta_2x_2^2+\beta_3x_3+\beta_4x_4$ | $y=-112.531+0.499x_1^2+10.913x_2^2+11.456x_3$ | 0.877 |
| 2b | $y=\beta_0+\beta_1x_1^2+\beta_2x_2^2+\beta_3x_3+\beta_4x_4$ | $y=-108.878+0.344x_1^2+10.571x_2^2+13.628x_3$ | 0.883 |
| 3a | $y=\beta_0+\beta_1x_1^3+\beta_2x_2^2+\beta_3x_3^2+\beta_4x_4$ | $y=-37.326+0.07282x_1^3+6.614x_2^2+0.788x_3^2$ | 0.836 |
| 3b | $y=\beta_0+\beta_1x_1^3+\beta_2x_2^2+\beta_3x_3^2+\beta_4x_4$ | $y=-53.507+0.03308x_1^3+8.655x_2^2+0.965x_3^2$ | 0.840 |
| 4a | $y=\dfrac{\beta_0x_1^{0.5\beta_1}e^{\beta_2x_2}\beta_3x_3}{\beta_4x_4}$ | $\ln y=-1.27+0.483\ln x_1+1.068x_2+0.816\ln x_3$ | 0.921 |
| 4b | $y=\dfrac{\beta_0x_1^{0.5\beta_1}e^{\beta_2x_2}\beta_3x_3}{\beta_4x_4}$ | $\ln y=-0.733+0.367\ln x_1+0.85x_2+1.039\ln x_3$ | 0.947 |
| 5a | $y=\beta_0x_1^{\beta_1}x_2^{\beta_2}x_3^{\beta_3}x_4^{\beta_4}$ | $\ln y=-1.365+0.45\ln x_1+2.778\ln x_2+1.053\ln x_3$ | 0.930 |
| 5b | $y=\beta_0x_1^{\beta_1}x_2^{\beta_2}x_3^{\beta_3}x_4^{\beta_4}$ | $\ln y=-1.829+0.488\ln x_1+2.914\ln x_2+1.164\ln x_3$ | 0.843 |
| 6a | $y=\beta_0+\beta_1x_1+\beta_2x_1x_2+\beta_3x_3+\beta_4x_4$ | $y=-15.047-20.907x_1+8.788x_1x_2+9.798x_3$ | 0.828 |
| 6b | $y=\beta_0+\beta_1x_1+\beta_2x_1x_2+\beta_3x_3+\beta_4x_4$ | $y=-23.202-23.101x_1+10.464x_1x_2+8.694x_3$ | 0.829 |
| 7a | $y=\beta_0+\beta_1x_1^2+\beta_2x_2^2x_3+\beta_3x_3+\beta_4x_4$ | $y=-11.289+0.572x_1^2+2.326x_2^2x_3-9.836x_3$ | 0.925 |
| 7b | $y=\beta_0+\beta_1x_1^2+\beta_2x_2^2x_3+\beta_3x_3+\beta_4x_4$ | $y=92.409+0.362x_1^2+1.797x_2^2x_3-31.876x_4$ | 0.933 |

| Model Number | Formulation | Stepwise Regression Model | Adjusted R-squared |
|---|---|---|---|
| 8a | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 + \beta_4 x_4$ | $y = -31.06 - 0.0904x_1^3 + 5.245x_1x_2 + 1.085x_3^2$ | 0.727 |
| 8b | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 + \beta_4 x_4$ | $y = -36.969 - 0.143x_1^3 + 6.731x_1x_2 + 0.972x_3^2$ | 0.800 |
| 9a | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3} e^{\beta_4 x_4}$ | $\ln y = 2.046 - 0.319x_1 + 0.151x_1x_2 + 0.219x_3$ | 0.855 |
| 9b | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3} e^{\beta_4 x_4}$ | $\ln y = 1.969 - 0.26x_1 + 0.139x_1x_2 + 0.201x_3$ | 0.853 |
| 10a | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3} x_4^{\beta_4}$ | $\ln y = 2.746 + 0.314\ln x_1 + 0.466x_3 \ln x_2 - 1.131\ln x_3$ | 0.797 |
| 10b | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3} x_4^{\beta_4}$ | $\ln y = 1.064 + 0.577\ln x_1 + 0.355x_3 \ln x_2$ | 0.900 |
| 11a | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_2 x_3 + \beta_4 x_3 + \beta_5 x_4$ | $y = 71.134 + 3.303x_1 7.975x_2x_3 - 13.361x_3 - 21.216x_4$ | 0.858 |
| 11b | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_2 x_3 + \beta_4 x_3 + \beta_5 x_4$ | $y = -26.59 + 3.293x_1 12.627x_2x_3 - 23.012x_3$ | 0.952 |
| 12a | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 x_3 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | $y = 97.13 + 0.301x_1^2 + 1.433x_2^2x_3 - 28.156x_4$ | 0.943 |
| 12b | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 x_3 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | $y = -34.739 + 0.625x_1^2 + 2.145x_2^2x_3 - 0.244x_3^2x_4$ | 0.879 |
| 13a | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | $y = 169.593 - 0.149x_1^3 + 6.68x_1x_2 + 0.335x_3^2x_4 - 54.78x_4$ | 0.778 |
| 13b | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | $y = -12.79 + 3.797x_1x_2$ | 0.581 |
| 14a | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3 x_4} e^{\beta_4 x_3} e^{\beta_5 x_4}$ | $\ln y = 2.032 - 0.291x_1 + 0.142x_1x_2 + 0.05836x_3x_4$ | 0.822 |
| 14b | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3 x_4} e^{\beta_4 x_3} e^{\beta_5 x_4}$ | $\ln y = 1.966 + 0.234x_1 + 0.129x_1x_2 + 0.206x_3$ | 0.813 |
| 15a | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3 x_4} x_4^{\beta_4}$ | $\ln y = 2.068 + 0.562\ln x_1 + 0.413x_3 \ln x_2 - 0.215x_4 \ln x_3$ | 0.863 |
| 15b | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3 x_4} x_4^{\beta_4}$ | $\ln y = 2.668 + 0.548\ln x_1 + 0.479x_3 \ln x_2 - 0.357x_4 \ln x_3$ | 0.786 |

| Function 3 | | | |
|---|---|---|---|
| **Model Number** | **Formulation** | **Stepwise Regression Model** | **Adjusted R-squared** |
| 1a | $y=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4$ | $y=-27.416+11.916x_1+11.496x_2$ | 0.984 |
| 1b | $y=\beta_0+\beta_1x_1+\beta_2x_2+\beta_3x_3+\beta_4x_4$ | $y=-47.648+11.779x_1+18.471x_2$ | 0.978 |
| 2a | $y=\beta_0+\beta_1x_1^2+\beta_2x_2^2+\beta_3x_3+\beta_4x_4$ | $y=16.134+0.987x_1^2+2.356x_2^2$ | 0.937 |
| 2b | $y=\beta_0+\beta_1x_1^2+\beta_2x_2^2+\beta_3x_3+\beta_4x_4$ | $y=5.696+1.008x_1^2+3.012x_2^2$ | 0.909 |
| 3a | $y=\beta_0+\beta_1x_1^3+\beta_2x_2^2+\beta_3x_3^2+\beta_4x_4$ | $y=22.996+0.115x_1^3+2.05x_2^2$ | 0.758 |
| 3b | $y=\beta_0+\beta_1x_1^3+\beta_2x_2^2+\beta_3x_3^2+\beta_4x_4$ | $y=21.665+0.104x_1^3+2.393x_2^2$ | 0.886 |
| 4a | $y=\dfrac{\beta_0x_1^{0.5\beta_1}e^{\beta_2x_2}\beta_3x_3}{\beta_4x_4}$ | $\ln y=1.887+0.929\ln x_1+0.276x_2$ | 0.984 |
| 4b | $y=\dfrac{\beta_0x_1^{0.5\beta_1}e^{\beta_2x_2}\beta_3x_3}{\beta_4x_4}$ | $\ln y=2.371+0.8565\ln x_1+0.15x_2$ | 0.977 |
| 5a | $y=\beta_0x_1^{\beta_1}x_2^{\beta_2}x_3^{\beta_3}x_4^{\beta_4}$ | $\ln y=2.152+0.908\ln x_1+0.555\ln x_2$ | 0.982 |
| 5b | $y=\beta_0x_1^{\beta_1}x_2^{\beta_2}x_3^{\beta_3}x_4^{\beta_4}$ | $\ln y=2.045+0.924\ln x_1+0.624\ln x_2$ | 0.981 |
| 6a | $y=\beta_0+\beta_1x_1+\beta_2x_1x_2+\beta_3x_3+\beta_4x_4$ | $y=8.958+4.552x_1+2.317x_1x_2$ | 0.985 |
| 6b | $y=\beta_0+\beta_1x_1+\beta_2x_1x_2+\beta_3x_3+\beta_4x_4$ | $y=2.173+5.002x_1+2.563x_1x_2$ | 0.982 |
| 7a | $y=\beta_0+\beta_1x_1^2+\beta_2x_2^2x_3+\beta_3x_3+\beta_4x_4$ | $y=34.752+0.934x_1^2$ | 0.886 |
| 7b | $y=\beta_0+\beta_1x_1^2+\beta_2x_2^2x_3+\beta_3x_3+\beta_4x_4$ | $y=35.402+0.997x_1^2$ | 0.908 |

| Model Number | Formulation | Stepwise Regression Model | Adjusted R-squared |
|---|---|---|---|
| 8a | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 + \beta_4 x_4$ | $y = 9.198 + 3.949 x_1 x_2$ | 0.968 |
| 8b | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 + \beta_4 x_4$ | $y = 12.802 + 0.0204 x_1^3 + 3.334 x_1 x_2$ | 0.982 |
| 9a | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3} e^{\beta_4 x_4}$ | $\ln y = 2.944 + 0.209 x_1$ | 0.895 |
| 9b | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3} e^{\beta_4 x_4}$ | $\ln y = 3.208 + 0.102 x_1 + 0.0257 x_1 x_2$ | 0.960 |
| 10a | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3} x_4^{\beta_4}$ | $\ln y = 2.624 + 0.891 \ln x_1 + 0.01954 x_3 \ln x_2$ | 0.972 |
| 10b | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3} x_4^{\beta_4}$ | $\ln y = 4.154 + 0.788 \ln x_1 - 0.924 x_3 \ln x_2$ | 0.957 |
| 11a | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_2 x_3 + \beta_4 x_3 + \beta_5 x_4$ | $y = 5.899 + 3.766 x_1 + 2.918 x_1 x_2$ | 0.990 |
| 11b | $y = \beta_0 + \beta_1 x_1 + \beta_2 x_1 x_2 + \beta_3 x_2 x_3 + \beta_4 x_3 + \beta_5 x_4$ | $y = 0.297 + 4.325 x_1 + 2.596 x_1 x_2 + 0.377 x_2 x_3$ | 0.984 |
| 12a | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 x_3 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | $y = 18.638 + 1.19 x_1^2 + 0.284 x_2^2 x_3$ | 0.945 |
| 12b | $y = \beta_0 + \beta_1 x_1^2 + \beta_2 x_2^2 x_3 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | $y = 27.614 + 0.956 x_1^2 + 0.444 x_2^2 x_3 - 0.0817 x_3^2 x_4$ | 0.951 |
| 13a | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | $y = 15.075 - 0.03657 x_1^3 + 2.936 x_1 x_2$ | 0.976 |
| 13b | $y = \beta_0 + \beta_1 x_1^3 + \beta_2 x_1 x_2 + \beta_3 x_3^2 x_4 + \beta_4 x_4$ | $y = 11.308 - 0.01623 x_1^3 + 3.464 x_1 x_2$ | 0.979 |
| 14a | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3 x_4} e^{\beta_4 x_3} e^{\beta_5 x_4}$ | $\ln y = 4.432 + 0.092 x_1 + 0.0281 x_1 x_2 - 0.299 x_4$ | 0.942 |
| 14b | $y = \beta_0 e^{\beta_1 x_1} e^{\beta_2 x_1 x_2} e^{\beta_3 x_3 x_4} e^{\beta_4 x_3} e^{\beta_5 x_4}$ | $\ln y = 3.052 + 0.07857 x_1 + 0.0434 x_1 x_2$ | 0.931 |
| 15a | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3 x_4} x_4^{\beta_4}$ | $\ln y = 2.621 + 0.891 \ln x_1 + 0.028 x_3 \ln x_2$ | 0.946 |
| 15b | $y = \beta_0 x_1^{\beta_1} x_2^{\beta_2 x_3} x_3^{\beta_3 x_4} x_4^{\beta_4}$ | $\ln y = 2.742 + 0.894 \ln x_1 + 0.0822 x_3 \ln x_2 - 0.0654 x_4 \ln x_3$ | 0.984 |

# APPENDIX D: ANN TRAINING AND TESTING DATA AND ESTIMATED Y-VALUES FOR FUNCTION 1

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 2.47 | 1.95 | 3.92 | 3.75 | 116.08 |
| 6.81 | 2.30 | 5.69 | 4.35 | 443.50 |
| 4.69 | 3.32 | 5.68 | 3.99 | 227.61 |
| 9.22 | 3.69 | 3.22 | 3.77 | 904.94 |
| 1.25 | 3.11 | 3.94 | 4.22 | 96.51 |
| 3.35 | 2.92 | 7.55 | 4.14 | 173.45 |
| 6.69 | 2.10 | 4.66 | 3.76 | 429.47 |
| 4.41 | 2.40 | 5.68 | 3.96 | 224.79 |
| 9.08 | 2.48 | 6.53 | 4.00 | 864.17 |
| 5.53 | 2.89 | 5.05 | 3.78 | 288.47 |
| 7.41 | 2.77 | 3.64 | 4.31 | 563.17 |
| 8.84 | 3.03 | 4.97 | 4.06 | 817.71 |
| 3.38 | 2.66 | 3.89 | 4.01 | 136.89 |
| 9.37 | 2.33 | 5.91 | 4.09 | 960.29 |
| 5.53 | 2.73 | 7.84 | 3.65 | 314.22 |
| 7.13 | 3.26 | 5.23 | 4.11 | 494.21 |
| 2.41 | 3.35 | 6.85 | 3.64 | 140.23 |
| 7.83 | 3.46 | 6.63 | 3.77 | 644.06 |
| 2.23 | 2.65 | 4.42 | 3.97 | 109.83 |
| 7.41 | 2.27 | 5.85 | 3.97 | 538.15 |
| 9.40 | 3.26 | 3.11 | 4.22 | 952.67 |
| 6.17 | 3.73 | 4.48 | 4.28 | 365.36 |
| 6.25 | 2.47 | 7.18 | 4.00 | 402.72 |
| 5.16 | 2.36 | 3.34 | 4.18 | 242.60 |
| 8.12 | 3.05 | 7.96 | 4.07 | 714.80 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 7.77 | 3.14 | 2.04 | 4.05 | 574.13 |
| 5.46 | 2.83 | 4.06 | 3.82 | 277.11 |
| 2.09 | 3.12 | 3.66 | 4.19 | 121.92 |
| 4.64 | 2.85 | 7.54 | 3.89 | 219.22 |
| 8.22 | 3.24 | 5.99 | 4.15 | 699.75 |
| 7.99 | 2.65 | 3.92 | 3.93 | 617.11 |
| 6.77 | 3.16 | 5.13 | 4.05 | 447.51 |
| 9.54 | 2.54 | 2.23 | 4.02 | 983.71 |
| 2.71 | 2.56 | 3.29 | 3.89 | 116.91 |
| 8.77 | 3.44 | 6.53 | 4.07 | 841.77 |
| 6.77 | 2.84 | 3.58 | 3.91 | 399.06 |
| 3.87 | 2.45 | 4.23 | 4.11 | 172.26 |
| 4.33 | 2.24 | 6.88 | 3.71 | 226.07 |
| 8.59 | 2.94 | 4.56 | 3.94 | 771.73 |
| 7.50 | 2.50 | 7.27 | 3.76 | 567.57 |
| 8.79 | 2.34 | 2.60 | 3.94 | 775.37 |
| 2.16 | 3.51 | 5.71 | 3.84 | 135.93 |
| 7.63 | 2.70 | 3.42 | 4.10 | 564.34 |
| 4.95 | 2.93 | 2.03 | 3.90 | 227.50 |
| 2.71 | 2.30 | 7.80 | 4.02 | 155.12 |
| 2.68 | 1.58 | 7.28 | 4.03 | 141.27 |
| 5.64 | 3.24 | 5.34 | 3.87 | 310.66 |
| 1.34 | 1.95 | 6.24 | 4.45 | 117.52 |
| 6.76 | 2.85 | 4.47 | 3.85 | 427.85 |
| 2.72 | 2.85 | 3.26 | 4.25 | 122.93 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 4.63 | 2.45 | 3.71 | 4.20 | 225.29 |
| 6.43 | 3.24 | 6.32 | 4.16 | 392.85 |
| 8.14 | 2.86 | 4.94 | 4.19 | 674.04 |
| 7.36 | 2.98 | 5.70 | 3.85 | 548.95 |
| 2.72 | 2.74 | 5.81 | 3.69 | 143.91 |
| 5.90 | 1.93 | 6.22 | 3.91 | 343.19 |
| 3.98 | 2.46 | 4.45 | 3.72 | 151.11 |
| 1.03 | 4.30 | 5.87 | 3.98 | 97.50 |
| 4.96 | 3.10 | 4.02 | 4.09 | 230.74 |
| 2.26 | 2.93 | 6.61 | 3.83 | 146.87 |
| 4.09 | 2.69 | 2.36 | 4.37 | 160.19 |
| 7.48 | 4.07 | 4.41 | 3.97 | 526.45 |
| 8.41 | 3.44 | 7.77 | 3.82 | 758.43 |
| 8.62 | 3.12 | 7.03 | 3.86 | 799.06 |
| 5.16 | 3.00 | 2.37 | 3.98 | 243.10 |
| 6.25 | 2.47 | 7.18 | 4.00 | 402.72 |
| 9.24 | 2.36 | 2.25 | 3.93 | 899.95 |
| 4.87 | 2.61 | 4.51 | 3.65 | 219.97 |
| 1.36 | 2.85 | 7.31 | 3.92 | 141.27 |
| 1.38 | 2.35 | 2.05 | 3.99 | 90.93 |
| 8.02 | 2.54 | 5.01 | 3.85 | 640.83 |
| 9.08 | 2.48 | 6.53 | 4.00 | 864.17 |
| 3.87 | 2.45 | 4.23 | 4.11 | 172.26 |
| 7.80 | 3.30 | 3.44 | 3.82 | 567.99 |
| 8.59 | 2.50 | 5.54 | 3.68 | 779.17 |

**Model a**

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 225.29 | 325.08 | 9957.82 | 98.78 |
| 392.85 | 437.00 | 1949.32 | |
| 674.04 | 638.71 | 1247.93 | |
| 548.95 | 574.37 | 646.21 | |
| 143.91 | 211.84 | 4613.98 | |
| 343.19 | 427.83 | 7164.34 | |
| 151.11 | 292.66 | 20037.71 | |
| 97.50 | 159.75 | 3874.58 | |
| 230.74 | 344.93 | 13039.66 | |
| 146.87 | 188.22 | 1710.13 | |
| 160.19 | 290.34 | 16937.79 | |
| 526.45 | 582.07 | 3093.70 | |
| 758.43 | 630.60 | 16340.29 | |
| 799.06 | 662.39 | 18678.40 | |
| 243.10 | 401.64 | 25135.66 | |
| 402.72 | 428.41 | 659.91 | |
| 899.95 | 760.33 | 19493.37 | |
| 219.97 | 365.80 | 21267.35 | |
| 141.27 | 165.73 | 598.52 | |
| 90.93 | 186.34 | 9103.19 | |
| 640.83 | 653.47 | 159.87 | |
| 864.17 | 703.82 | 25711.45 | |
| 172.26 | 268.74 | 9307.75 | |
| 567.99 | 649.93 | 6713.75 | |
| 779.17 | 698.71 | 6473.55 | |

**Model b**

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 225.29 | 312.68 | 7636.28 | 119.42 |
| 392.85 | 415.72 | 522.83 | |
| 674.04 | 584.88 | 7949.02 | |
| 548.95 | 516.28 | 1067.33 | |
| 143.91 | 219.75 | 5751.76 | |
| 343.19 | 362.96 | 390.76 | |
| 151.11 | 273.29 | 14927.88 | |
| 97.50 | 193.62 | 9239.81 | |
| 230.74 | 338.79 | 11674.82 | |
| 146.87 | 206.66 | 3574.86 | |
| 160.19 | 297.63 | 18890.54 | |
| 526.45 | 561.03 | 1196.06 | |
| 758.43 | 569.75 | 35600.25 | |
| 799.06 | 593.86 | 42105.79 | |
| 243.10 | 385.92 | 20397.05 | |
| 402.72 | 377.61 | 630.35 | |
| 899.95 | 680.31 | 48240.88 | |
| 219.97 | 327.63 | 11589.89 | |
| 141.27 | 191.96 | 2569.46 | |
| 90.93 | 206.56 | 13369.61 | |
| 640.83 | 578.01 | 3946.55 | |
| 864.17 | 623.76 | 57796.69 | |
| 172.26 | 265.67 | 8726.35 | |
| 567.99 | 596.25 | 798.47 | |
| 779.17 | 612.01 | 27941.33 | |

**Average**
**RMSE:** 109.10

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 6.69 | 2.10 | 4.66 | 3.76 | 429.47 |
| 7.02 | 2.62 | 7.56 | 4.09 | 488.64 |
| 1.71 | 3.26 | 3.05 | 3.79 | 83.17 |
| 6.24 | 2.50 | 7.89 | 4.16 | 413.48 |
| 8.22 | 3.24 | 5.99 | 4.15 | 699.75 |
| 5.81 | 2.42 | 4.31 | 3.63 | 321.51 |
| 3.39 | 3.12 | 5.69 | 3.77 | 165.89 |
| 8.77 | 2.57 | 2.94 | 4.14 | 783.10 |
| 7.64 | 2.57 | 5.99 | 4.33 | 569.96 |
| 1.77 | 2.45 | 2.18 | 3.91 | 76.79 |
| 7.77 | 3.14 | 2.04 | 4.05 | 574.13 |
| 6.33 | 2.00 | 4.49 | 3.71 | 360.77 |
| 6.07 | 1.44 | 3.04 | 4.19 | 327.23 |
| 7.21 | 3.20 | 3.49 | 3.95 | 489.68 |
| 7.13 | 3.26 | 5.23 | 4.11 | 494.21 |
| 2.81 | 2.05 | 7.65 | 3.96 | 174.24 |
| 7.52 | 2.27 | 4.23 | 3.86 | 535.36 |
| 8.66 | 2.03 | 3.39 | 4.01 | 750.85 |
| 5.24 | 2.95 | 6.34 | 3.65 | 272.98 |
| 9.53 | 2.20 | 4.82 | 4.22 | 978.95 |
| 6.81 | 2.36 | 2.02 | 3.98 | 429.73 |
| 9.19 | 2.09 | 3.92 | 3.71 | 896.59 |
| 4.93 | 3.19 | 7.27 | 4.07 | 275.68 |
| 9.58 | 2.61 | 2.89 | 3.50 | 977.06 |
| 9.58 | 2.47 | 5.33 | 3.63 | 1002.07 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 4.13 | 3.22 | 4.98 | 4.19 | 185.69 |
| 9.08 | 2.48 | 6.53 | 4.00 | 864.17 |
| 1.34 | 1.95 | 6.24 | 4.45 | 117.52 |
| 8.92 | 3.78 | 4.50 | 4.00 | 845.92 |
| 7.45 | 2.91 | 7.35 | 3.84 | 558.55 |
| 4.69 | 2.17 | 2.64 | 3.90 | 211.47 |
| 8.02 | 2.54 | 5.01 | 3.85 | 640.83 |
| 9.13 | 2.33 | 2.08 | 4.09 | 860.20 |
| 1.38 | 2.35 | 2.05 | 3.99 | 90.93 |
| 7.21 | 3.20 | 3.49 | 3.95 | 489.68 |
| 8.62 | 2.39 | 3.45 | 4.51 | 760.18 |
| 9.17 | 2.28 | 5.21 | 4.09 | 893.12 |
| 3.43 | 3.49 | 2.68 | 3.78 | 147.47 |
| 4.47 | 3.30 | 7.35 | 3.83 | 248.68 |
| 8.13 | 2.95 | 5.61 | 3.87 | 676.06 |
| 1.88 | 3.33 | 2.09 | 3.82 | 85.69 |
| 5.94 | 2.92 | 5.02 | 4.02 | 316.19 |
| 9.65 | 2.82 | 4.57 | 3.89 | 1006.50 |
| 5.81 | 2.42 | 4.31 | 3.63 | 321.51 |
| 8.08 | 3.38 | 2.91 | 3.94 | 636.97 |
| 6.35 | 2.25 | 4.61 | 4.38 | 392.80 |
| 7.79 | 2.13 | 5.45 | 3.80 | 596.56 |
| 7.92 | 2.24 | 3.47 | 3.65 | 602.92 |
| 7.13 | 2.53 | 7.88 | 3.89 | 517.70 |
| 5.86 | 2.86 | 4.45 | 3.80 | 311.17 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 6.09 | 2.80 | 2.54 | 3.94 | 324.80 |
| 6.84 | 2.61 | 4.66 | 4.09 | 427.61 |
| 4.69 | 2.73 | 2.26 | 4.00 | 199.51 |
| 5.84 | 2.76 | 6.85 | 3.93 | 341.32 |
| 6.24 | 2.72 | 3.59 | 3.87 | 337.23 |
| 2.03 | 2.41 | 6.88 | 3.80 | 128.79 |
| 9.98 | 2.42 | 2.72 | 4.24 | 1109.20 |
| 6.44 | 2.58 | 7.46 | 3.71 | 413.33 |
| 1.81 | 2.45 | 5.29 | 3.99 | 104.23 |
| 6.77 | 3.16 | 5.13 | 4.05 | 447.51 |
| 7.76 | 2.58 | 7.95 | 3.99 | 619.75 |
| 8.57 | 2.74 | 4.38 | 3.87 | 750.31 |
| 2.43 | 2.48 | 7.42 | 3.77 | 147.10 |
| 4.64 | 2.85 | 7.54 | 3.89 | 219.22 |
| 8.92 | 3.78 | 4.50 | 4.00 | 845.92 |
| 9.58 | 2.47 | 5.33 | 3.63 | 1002.07 |
| 1.23 | 2.93 | 3.59 | 4.04 | 92.39 |
| 1.12 | 2.98 | 5.69 | 4.22 | 119.86 |
| 8.21 | 3.37 | 3.97 | 4.37 | 681.74 |
| 1.03 | 2.95 | 6.41 | 3.96 | 122.22 |
| 4.18 | 2.69 | 3.25 | 3.83 | 150.17 |
| 4.16 | 2.39 | 2.85 | 4.13 | 169.88 |
| 1.10 | 3.11 | 2.59 | 4.17 | 103.75 |
| 6.39 | 1.83 | 5.54 | 3.76 | 379.46 |
| 5.91 | 2.78 | 4.61 | 3.90 | 347.85 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 324.80 | 467.23 | 20286.95 | 123.63 |
| 427.61 | 495.94 | 4669.21 | |
| 199.51 | 340.13 | 19773.99 | |
| 341.32 | 363.17 | 477.53 | |
| 337.23 | 473.35 | 18529.53 | |
| 128.79 | 177.24 | 2347.28 | |
| 1109.20 | 778.84 | 109140.18 | |
| 413.33 | 439.94 | 708.28 | |
| 104.23 | 174.72 | 4969.28 | |
| 447.51 | 448.09 | 0.34 | |
| 619.75 | 534.07 | 7341.13 | |
| 750.31 | 678.64 | 5135.96 | |
| 147.10 | 185.02 | 1437.61 | |
| 219.22 | 264.11 | 2014.77 | |
| 845.92 | 639.43 | 42636.55 | |
| 1002.07 | 760.83 | 58197.94 | |
| 92.39 | 162.69 | 4941.60 | |
| 119.86 | 151.04 | 971.91 | |
| 681.74 | 569.76 | 12539.47 | |
| 122.22 | 150.79 | 816.06 | |
| 150.17 | 300.75 | 22673.43 | |
| 169.88 | 296.73 | 16091.61 | |
| 103.75 | 160.20 | 3186.52 | |
| 379.46 | 517.25 | 18987.42 | |
| 347.85 | 413.07 | 4253.74 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 324.80 | 440.72 | 13438.14 | 135.11 |
| 427.61 | 519.50 | 8444.10 | |
| 199.51 | 330.05 | 17040.40 | |
| 341.32 | 429.84 | 7836.59 | |
| 337.23 | 460.85 | 15281.78 | |
| 128.79 | 200.64 | 5163.07 | |
| 1109.20 | 742.87 | 134200.81 | |
| 413.33 | 493.43 | 6416.02 | |
| 104.23 | 190.61 | 7462.37 | |
| 447.51 | 485.97 | 1479.10 | |
| 619.75 | 602.78 | 288.12 | |
| 750.31 | 651.68 | 9727.41 | |
| 147.10 | 215.70 | 4705.79 | |
| 219.22 | 330.69 | 12425.09 | |
| 845.92 | 631.41 | 46014.65 | |
| 1002.07 | 723.60 | 77545.01 | |
| 92.39 | 165.76 | 5382.53 | |
| 119.86 | 164.19 | 1965.52 | |
| 681.74 | 591.99 | 8054.55 | |
| 122.22 | 162.33 | 1609.03 | |
| 150.17 | 298.66 | 22049.97 | |
| 169.88 | 307.09 | 18825.69 | |
| 103.75 | 160.57 | 3228.42 | |
| 379.46 | 523.88 | 20856.15 | |
| 347.85 | 431.08 | 6926.79 | |

Average
RMSE: 129.37

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 2.80 | 2.61 | 4.25 | 3.88 | 115.62 |
| 4.71 | 3.00 | 5.60 | 4.04 | 215.92 |
| 1.00 | 2.16 | 3.47 | 4.39 | 106.17 |
| 9.94 | 3.42 | 2.39 | 3.99 | 1090.88 |
| 5.64 | 3.85 | 4.06 | 3.89 | 295.97 |
| 2.40 | 2.89 | 3.99 | 3.74 | 106.42 |
| 9.44 | 2.98 | 5.74 | 3.89 | 979.19 |
| 3.87 | 2.45 | 4.23 | 4.11 | 172.26 |
| 2.62 | 2.80 | 4.72 | 4.13 | 113.92 |
| 2.81 | 3.00 | 2.76 | 3.85 | 133.15 |
| 5.65 | 1.92 | 7.79 | 3.87 | 329.69 |
| 6.88 | 3.69 | 6.42 | 4.11 | 499.42 |
| 7.14 | 2.48 | 3.67 | 3.92 | 478.04 |
| 6.76 | 3.08 | 7.54 | 4.40 | 475.07 |
| 5.64 | 3.24 | 5.34 | 3.87 | 310.66 |
| 3.01 | 3.02 | 7.88 | 4.01 | 181.21 |
| 3.81 | 4.02 | 6.65 | 4.07 | 187.34 |
| 7.92 | 2.87 | 7.65 | 3.75 | 652.28 |
| 3.37 | 2.36 | 6.35 | 3.88 | 159.26 |
| 9.42 | 3.31 | 5.79 | 4.04 | 967.23 |
| 7.52 | 2.27 | 4.23 | 3.86 | 535.36 |
| 7.85 | 3.27 | 5.56 | 4.08 | 604.44 |
| 9.53 | 3.01 | 3.22 | 3.98 | 988.66 |
| 8.39 | 2.22 | 7.84 | 4.14 | 742.67 |
| 3.66 | 2.61 | 3.81 | 3.99 | 150.07 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 9.73 | 3.09 | 7.87 | 3.78 | 1080.12 |
| 8.21 | 3.37 | 3.97 | 4.37 | 681.74 |
| 1.35 | 2.50 | 4.74 | 3.91 | 102.81 |
| 6.94 | 2.99 | 5.65 | 4.13 | 458.89 |
| 9.75 | 2.09 | 4.58 | 4.14 | 1059.86 |
| 4.13 | 3.22 | 4.98 | 4.19 | 185.69 |
| 8.66 | 2.03 | 3.39 | 4.01 | 750.85 |
| 2.16 | 3.51 | 5.71 | 3.84 | 135.93 |
| 7.02 | 2.62 | 7.56 | 4.09 | 488.64 |
| 2.56 | 3.17 | 7.54 | 4.15 | 155.10 |
| 3.75 | 2.45 | 3.44 | 4.04 | 168.78 |
| 3.64 | 3.30 | 3.46 | 4.08 | 171.26 |
| 5.85 | 3.31 | 3.00 | 4.02 | 311.22 |
| 7.79 | 2.13 | 5.45 | 3.80 | 596.56 |
| 7.17 | 2.93 | 5.61 | 4.04 | 505.13 |
| 7.67 | 2.45 | 5.98 | 3.73 | 579.69 |
| 5.34 | 2.61 | 2.83 | 4.09 | 256.90 |
| 9.80 | 2.22 | 2.79 | 4.27 | 1070.64 |
| 5.85 | 2.13 | 5.26 | 3.92 | 312.96 |
| 5.53 | 2.89 | 5.05 | 3.78 | 288.47 |
| 8.90 | 2.79 | 2.94 | 3.96 | 812.38 |
| 4.40 | 3.31 | 6.71 | 4.16 | 219.63 |
| 6.48 | 3.75 | 2.15 | 4.09 | 370.25 |
| 5.24 | 2.95 | 6.34 | 3.65 | 272.98 |
| 2.26 | 2.93 | 6.61 | 3.83 | 146.87 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 5.64 | 2.04 | 6.58 | 4.11 | 338.63 |
| 2.68 | 1.58 | 7.28 | 4.03 | 141.27 |
| 5.90 | 1.93 | 6.22 | 3.91 | 343.19 |
| 2.69 | 3.63 | 5.60 | 4.15 | 144.00 |
| 7.44 | 3.41 | 3.78 | 3.68 | 552.02 |
| 3.93 | 2.58 | 3.12 | 3.79 | 173.29 |
| 5.81 | 2.42 | 4.31 | 3.63 | 321.51 |
| 5.13 | 2.97 | 2.25 | 3.97 | 232.56 |
| 4.53 | 2.69 | 2.73 | 4.17 | 189.20 |
| 5.06 | 3.16 | 2.36 | 4.07 | 240.02 |
| 6.49 | 3.15 | 2.72 | 3.68 | 360.99 |
| 2.30 | 2.83 | 5.23 | 3.86 | 129.63 |
| 5.84 | 2.76 | 6.85 | 3.93 | 341.32 |
| 9.73 | 3.09 | 7.87 | 3.78 | 1080.12 |
| 2.40 | 2.89 | 3.99 | 3.74 | 106.42 |
| 2.50 | 2.64 | 6.03 | 4.15 | 140.27 |
| 2.62 | 2.80 | 4.72 | 4.13 | 113.92 |
| 5.16 | 2.36 | 3.34 | 4.18 | 242.60 |
| 4.58 | 2.70 | 4.58 | 4.02 | 209.27 |
| 8.68 | 2.75 | 6.95 | 3.77 | 794.84 |
| 6.22 | 2.14 | 5.46 | 4.10 | 349.88 |
| 3.53 | 4.04 | 7.01 | 4.16 | 180.45 |
| 1.12 | 2.98 | 5.69 | 4.22 | 119.86 |
| 5.26 | 2.88 | 3.34 | 4.14 | 262.00 |
| 9.26 | 2.25 | 4.46 | 4.32 | 918.17 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 338.63 | 383.73 | 2034.04 | 130.20 |
| 141.27 | 196.08 | 3003.95 | |
| 343.19 | 421.18 | 6081.78 | |
| 144.00 | 213.90 | 4885.36 | |
| 552.02 | 633.38 | 6619.92 | |
| 173.29 | 305.10 | 17373.57 | |
| 321.51 | 458.15 | 18671.12 | |
| 232.56 | 411.02 | 31847.95 | |
| 189.20 | 339.96 | 22729.55 | |
| 240.02 | 400.56 | 25773.55 | |
| 360.99 | 552.95 | 36849.28 | |
| 129.63 | 202.94 | 5374.55 | |
| 341.32 | 417.12 | 5745.29 | |
| 1080.12 | 766.75 | 98201.86 | |
| 106.42 | 217.15 | 12261.58 | |
| 140.27 | 198.50 | 3390.86 | |
| 113.92 | 211.22 | 9466.55 | |
| 242.60 | 382.11 | 19463.11 | |
| 209.27 | 330.16 | 14614.22 | |
| 794.84 | 694.58 | 10052.72 | |
| 349.88 | 455.29 | 11111.71 | |
| 180.45 | 246.19 | 4321.50 | |
| 119.86 | 163.20 | 1878.35 | |
| 262.00 | 399.62 | 18938.09 | |
| 918.17 | 736.26 | 33091.90 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 338.63 | 431.97 | 8712.30 | 124.19 |
| 141.27 | 249.54 | 11721.99 | |
| 343.19 | 493.93 | 22721.71 | |
| 144.00 | 163.70 | 388.03 | |
| 552.02 | 490.88 | 3737.92 | |
| 173.29 | 287.08 | 12949.00 | |
| 321.51 | 464.03 | 20313.32 | |
| 232.56 | 336.03 | 10706.12 | |
| 189.20 | 304.66 | 13331.22 | |
| 240.02 | 303.33 | 4008.50 | |
| 360.99 | 448.94 | 7735.38 | |
| 129.63 | 182.02 | 2744.62 | |
| 341.32 | 362.85 | 463.39 | |
| 1080.12 | 691.55 | 150987.31 | |
| 106.42 | 189.26 | 6862.14 | |
| 140.27 | 187.87 | 2265.84 | |
| 113.92 | 190.36 | 5842.93 | |
| 242.60 | 387.54 | 21006.46 | |
| 209.27 | 294.25 | 7221.28 | |
| 794.84 | 654.35 | 19737.90 | |
| 349.88 | 495.66 | 21252.66 | |
| 180.45 | 166.66 | 190.26 | |
| 119.86 | 151.39 | 994.39 | |
| 262.00 | 334.08 | 5195.56 | |
| 918.17 | 761.71 | 24480.27 | |

Average
RMSE: 127.19

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 2.26 | 3.22 | 2.91 | 4.06 | 97.67 |
| 7.50 | 2.50 | 7.27 | 3.76 | 567.57 |
| 2.51 | 3.25 | 3.76 | 4.01 | 140.73 |
| 5.94 | 2.92 | 5.02 | 4.02 | 316.19 |
| 5.16 | 2.36 | 3.34 | 4.18 | 242.60 |
| 7.83 | 3.46 | 6.63 | 3.77 | 644.06 |
| 1.70 | 2.86 | 4.19 | 4.13 | 91.37 |
| 7.63 | 2.07 | 4.49 | 4.21 | 560.03 |
| 3.71 | 2.65 | 3.12 | 4.31 | 149.62 |
| 8.39 | 2.22 | 7.84 | 4.14 | 742.67 |
| 4.18 | 3.26 | 6.52 | 3.66 | 218.27 |
| 2.71 | 2.56 | 3.29 | 3.89 | 116.91 |
| 7.52 | 2.27 | 4.23 | 3.86 | 535.36 |
| 9.34 | 2.41 | 3.80 | 4.06 | 925.54 |
| 6.36 | 3.12 | 7.46 | 3.98 | 405.95 |
| 1.03 | 2.71 | 7.57 | 4.01 | 139.51 |
| 4.14 | 3.24 | 6.01 | 4.00 | 204.42 |
| 7.65 | 2.96 | 4.69 | 4.07 | 550.36 |
| 4.53 | 2.69 | 2.73 | 4.17 | 189.20 |
| 6.05 | 2.43 | 7.73 | 3.89 | 377.36 |
| 1.37 | 2.72 | 4.43 | 4.18 | 120.06 |
| 3.37 | 3.92 | 2.79 | 3.97 | 138.06 |
| 9.17 | 2.28 | 5.21 | 4.09 | 893.12 |
| 8.49 | 2.91 | 6.83 | 3.98 | 747.72 |
| 1.50 | 3.21 | 5.39 | 4.03 | 120.95 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.71 | 3.26 | 3.05 | 3.79 | 83.17 |
| 8.27 | 3.16 | 7.77 | 4.05 | 724.34 |
| 6.69 | 2.10 | 4.66 | 3.76 | 429.47 |
| 7.19 | 3.42 | 2.06 | 3.85 | 469.72 |
| 7.41 | 2.27 | 5.85 | 3.97 | 538.15 |
| 1.22 | 2.12 | 7.94 | 4.05 | 152.47 |
| 8.21 | 3.37 | 3.97 | 4.37 | 681.74 |
| 7.46 | 3.17 | 3.10 | 3.98 | 527.84 |
| 7.20 | 3.42 | 5.54 | 4.20 | 526.04 |
| 9.50 | 2.85 | 6.98 | 3.88 | 1012.81 |
| 2.81 | 3.00 | 2.76 | 3.85 | 133.15 |
| 5.95 | 2.61 | 3.14 | 3.89 | 304.78 |
| 8.32 | 3.42 | 4.42 | 4.26 | 696.90 |
| 8.09 | 3.55 | 5.27 | 3.79 | 656.89 |
| 1.94 | 2.46 | 3.15 | 3.97 | 87.02 |
| 7.80 | 3.30 | 3.44 | 3.82 | 567.99 |
| 5.56 | 2.34 | 5.59 | 4.11 | 295.21 |
| 5.99 | 1.96 | 6.13 | 3.66 | 326.85 |
| 8.10 | 3.33 | 3.27 | 4.17 | 656.30 |
| 6.17 | 3.73 | 4.48 | 4.28 | 365.36 |
| 2.47 | 1.95 | 3.92 | 3.75 | 116.08 |
| 5.93 | 4.10 | 4.77 | 3.75 | 326.03 |
| 8.19 | 3.21 | 2.35 | 3.93 | 640.93 |
| 1.09 | 3.19 | 5.70 | 3.60 | 104.45 |
| 2.36 | 2.62 | 2.93 | 4.16 | 130.54 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 2.43 | 2.48 | 7.42 | 3.77 | 147.10 |
| 2.53 | 2.84 | 3.62 | 3.86 | 118.78 |
| 2.45 | 3.32 | 4.65 | 4.00 | 120.87 |
| 3.53 | 4.04 | 7.01 | 4.16 | 180.45 |
| 3.35 | 2.92 | 7.55 | 4.14 | 173.45 |
| 5.81 | 2.42 | 4.31 | 3.63 | 321.51 |
| 7.79 | 2.13 | 5.45 | 3.80 | 596.56 |
| 2.27 | 3.46 | 2.07 | 4.31 | 102.08 |
| 5.42 | 2.91 | 2.12 | 4.17 | 263.19 |
| 6.22 | 2.14 | 5.46 | 4.10 | 349.88 |
| 4.05 | 2.40 | 6.34 | 3.84 | 192.66 |
| 7.64 | 2.57 | 5.99 | 4.33 | 569.96 |
| 5.16 | 3.00 | 2.37 | 3.98 | 243.10 |
| 8.73 | 2.09 | 5.30 | 4.19 | 772.98 |
| 4.64 | 2.85 | 7.54 | 3.89 | 219.22 |
| 7.20 | 3.42 | 5.54 | 4.20 | 526.04 |
| 5.65 | 2.10 | 5.47 | 3.74 | 311.22 |
| 7.45 | 2.70 | 4.22 | 4.18 | 536.13 |
| 5.95 | 2.61 | 3.14 | 3.89 | 304.78 |
| 3.75 | 2.45 | 3.44 | 4.04 | 168.78 |
| 2.33 | 3.87 | 6.80 | 3.77 | 144.55 |
| 1.58 | 3.82 | 5.03 | 4.27 | 137.81 |
| 9.16 | 2.58 | 6.61 | 4.13 | 905.39 |
| 5.64 | 3.85 | 4.06 | 3.89 | 295.97 |
| 4.20 | 2.94 | 2.72 | 4.22 | 168.33 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 147.10 | 290.94 | 20690.85 | 122.77 |
| 118.78 | 206.40 | 7676.48 | |
| 120.87 | 180.42 | 3546.62 | |
| 180.45 | 176.71 | 13.98 | |
| 173.45 | 237.88 | 4151.08 | |
| 321.51 | 526.41 | 41983.77 | |
| 596.56 | 662.66 | 4369.85 | |
| 102.08 | 159.10 | 3251.47 | |
| 263.19 | 261.68 | 2.27 | |
| 349.88 | 524.89 | 30629.98 | |
| 192.66 | 382.86 | 36175.19 | |
| 569.96 | 523.73 | 2137.19 | |
| 243.10 | 269.87 | 716.44 | |
| 772.98 | 657.43 | 13352.65 | |
| 219.22 | 367.82 | 22083.31 | |
| 526.04 | 360.49 | 27407.52 | |
| 311.22 | 564.32 | 64060.45 | |
| 536.13 | 482.49 | 2877.00 | |
| 304.78 | 418.45 | 12919.85 | |
| 168.78 | 264.02 | 9070.55 | |
| 144.55 | 180.61 | 1300.34 | |
| 137.81 | 156.11 | 334.86 | |
| 905.39 | 654.08 | 63157.02 | |
| 295.97 | 240.74 | 3050.26 | |
| 168.33 | 211.51 | 1864.94 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 147.10 | 230.93 | 7026.99 | 126.14 |
| 118.78 | 228.77 | 12098.10 | |
| 120.87 | 249.33 | 16501.29 | |
| 180.45 | 353.85 | 30068.24 | |
| 173.45 | 303.39 | 16884.77 | |
| 321.51 | 353.19 | 1003.63 | |
| 596.56 | 489.18 | 11530.53 | |
| 102.08 | 248.11 | 21325.38 | |
| 263.19 | 374.91 | 12482.10 | |
| 349.88 | 419.09 | 4789.92 | |
| 192.66 | 289.47 | 9371.34 | |
| 569.96 | 540.44 | 871.42 | |
| 243.10 | 349.59 | 11339.67 | |
| 772.98 | 560.26 | 45248.75 | |
| 219.22 | 353.59 | 18054.93 | |
| 526.04 | 536.24 | 104.06 | |
| 311.22 | 350.16 | 1516.49 | |
| 536.13 | 511.22 | 620.26 | |
| 304.78 | 381.15 | 5832.08 | |
| 168.78 | 269.75 | 10194.99 | |
| 144.55 | 260.19 | 13371.53 | |
| 137.81 | 247.32 | 11992.26 | |
| 905.39 | 593.39 | 97345.75 | |
| 295.97 | 425.70 | 16830.45 | |
| 168.33 | 314.62 | 21401.82 | |

Average
RMSE: 124.46

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.03 | 2.71 | 7.57 | 4.01 | 139.51 |
| 1.80 | 3.60 | 7.93 | 4.06 | 161.77 |
| 3.53 | 4.04 | 7.01 | 4.16 | 180.45 |
| 7.47 | 2.57 | 3.13 | 4.23 | 529.17 |
| 7.36 | 2.98 | 5.70 | 3.85 | 548.95 |
| 6.01 | 1.91 | 2.78 | 3.64 | 315.36 |
| 6.53 | 2.94 | 7.44 | 3.79 | 419.52 |
| 4.53 | 2.69 | 2.73 | 4.17 | 189.20 |
| 3.71 | 2.65 | 3.12 | 4.31 | 149.62 |
| 7.79 | 2.13 | 5.45 | 3.80 | 596.56 |
| 8.24 | 3.52 | 4.63 | 3.84 | 694.28 |
| 6.16 | 2.56 | 6.60 | 3.95 | 388.12 |
| 5.32 | 2.94 | 4.26 | 4.11 | 252.90 |
| 8.77 | 2.57 | 2.94 | 4.14 | 783.10 |
| 6.48 | 3.75 | 2.15 | 4.09 | 370.25 |
| 5.65 | 1.92 | 7.79 | 3.87 | 329.69 |
| 6.81 | 2.30 | 5.69 | 4.35 | 443.50 |
| 7.38 | 2.92 | 3.04 | 3.93 | 514.12 |
| 5.51 | 3.00 | 2.38 | 4.08 | 281.92 |
| 2.59 | 2.54 | 2.57 | 4.08 | 107.74 |
| 9.94 | 3.42 | 2.39 | 3.99 | 1090.88 |
| 5.53 | 2.89 | 5.05 | 3.78 | 288.47 |
| 7.80 | 3.20 | 7.46 | 3.92 | 610.80 |
| 4.14 | 3.24 | 6.01 | 4.00 | 204.42 |
| 9.46 | 3.59 | 7.02 | 3.80 | 1003.16 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 3.81 | 2.39 | 6.20 | 3.68 | 185.03 |
| 1.63 | 1.81 | 7.10 | 3.99 | 150.85 |
| 7.20 | 3.42 | 5.54 | 4.20 | 526.04 |
| 8.09 | 3.55 | 5.27 | 3.79 | 656.89 |
| 1.02 | 2.28 | 2.60 | 4.15 | 86.61 |
| 5.80 | 3.19 | 6.18 | 4.28 | 320.89 |
| 8.39 | 2.22 | 7.84 | 4.14 | 742.67 |
| 5.76 | 3.10 | 7.63 | 3.67 | 349.75 |
| 8.59 | 2.50 | 5.54 | 3.68 | 779.17 |
| 6.31 | 3.43 | 3.22 | 4.04 | 361.47 |
| 5.10 | 2.98 | 6.32 | 4.16 | 265.46 |
| 9.44 | 2.70 | 7.94 | 3.74 | 1009.36 |
| 1.54 | 2.50 | 5.73 | 4.00 | 135.15 |
| 2.22 | 2.67 | 4.56 | 3.99 | 135.65 |
| 6.86 | 2.48 | 7.98 | 3.90 | 485.61 |
| 4.89 | 1.97 | 6.99 | 3.91 | 262.07 |
| 5.86 | 2.86 | 4.45 | 3.80 | 311.17 |
| 2.58 | 2.40 | 3.15 | 3.77 | 86.91 |
| 6.69 | 2.46 | 3.20 | 4.13 | 424.55 |
| 4.04 | 2.58 | 7.67 | 3.99 | 215.84 |
| 3.75 | 2.45 | 3.44 | 4.04 | 168.78 |
| 5.50 | 2.75 | 6.77 | 4.18 | 308.08 |
| 7.60 | 3.21 | 5.00 | 4.10 | 585.09 |
| 1.43 | 1.45 | 3.72 | 4.13 | 111.58 |
| 1.87 | 1.86 | 3.57 | 3.96 | 101.99 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 5.50 | 2.75 | 6.77 | 4.18 | 308.08 |
| 5.69 | 2.05 | 2.27 | 3.94 | 271.61 |
| 9.53 | 3.01 | 3.22 | 3.98 | 988.66 |
| 9.75 | 2.09 | 4.58 | 4.14 | 1059.86 |
| 2.37 | 2.93 | 7.66 | 3.96 | 155.45 |
| 2.70 | 3.47 | 6.47 | 4.19 | 171.87 |
| 3.20 | 2.21 | 2.79 | 3.67 | 127.84 |
| 2.09 | 3.12 | 3.66 | 4.19 | 121.92 |
| 8.02 | 2.54 | 5.01 | 3.85 | 640.83 |
| 1.03 | 2.95 | 6.41 | 3.96 | 122.22 |
| 8.12 | 2.87 | 5.95 | 4.14 | 672.92 |
| 7.76 | 2.58 | 7.95 | 3.99 | 619.75 |
| 7.69 | 2.70 | 7.55 | 4.03 | 629.76 |
| 3.39 | 3.12 | 5.69 | 3.77 | 165.89 |
| 5.96 | 2.43 | 3.68 | 4.31 | 336.86 |
| 4.69 | 2.17 | 2.64 | 3.90 | 211.47 |
| 5.81 | 2.42 | 4.31 | 3.63 | 321.51 |
| 4.81 | 3.20 | 2.97 | 4.26 | 212.89 |
| 7.48 | 4.07 | 4.41 | 3.97 | 526.45 |
| 2.36 | 2.62 | 2.93 | 4.16 | 130.54 |
| 2.72 | 2.74 | 5.81 | 3.69 | 143.91 |
| 5.53 | 2.73 | 7.84 | 3.65 | 314.22 |
| 6.81 | 2.36 | 2.02 | 3.98 | 429.73 |
| 8.98 | 3.20 | 2.23 | 4.15 | 845.34 |
| 4.14 | 3.24 | 6.01 | 4.00 | 204.42 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 308.08 | 304.33 | 14.09 | 147.72 |
| 271.61 | 415.09 | 20587.33 | |
| 988.66 | 711.83 | 76634.35 | |
| 1059.86 | 641.50 | 175025.33 | |
| 155.45 | 209.19 | 2887.87 | |
| 171.87 | 207.64 | 1279.25 | |
| 127.84 | 313.33 | 34405.16 | |
| 121.92 | 200.64 | 6197.55 | |
| 640.83 | 609.83 | 960.85 | |
| 122.22 | 185.37 | 3987.46 | |
| 672.92 | 526.81 | 21347.02 | |
| 619.75 | 503.62 | 13485.59 | |
| 629.76 | 495.80 | 17944.04 | |
| 165.89 | 296.76 | 17127.95 | |
| 336.86 | 325.70 | 124.46 | |
| 211.47 | 351.99 | 19746.40 | |
| 321.51 | 507.99 | 34775.20 | |
| 212.89 | 298.90 | 7398.28 | |
| 526.45 | 598.51 | 5192.48 | |
| 130.54 | 206.98 | 5843.41 | |
| 143.91 | 267.72 | 15329.49 | |
| 314.22 | 437.67 | 15240.09 | |
| 429.73 | 509.82 | 6414.58 | |
| 845.34 | 654.31 | 36492.99 | |
| 204.42 | 288.75 | 7112.27 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 308.08 | 335.03 | 726.53 | 174.57 |
| 271.61 | 305.27 | 1132.78 | |
| 988.66 | 566.48 | 178233.52 | |
| 1059.86 | 544.71 | 265382.13 | |
| 155.45 | 245.09 | 8035.85 | |
| 171.87 | 223.34 | 2648.73 | |
| 127.84 | 246.29 | 14030.95 | |
| 121.92 | 180.60 | 3443.06 | |
| 640.83 | 532.99 | 11630.08 | |
| 122.22 | 197.45 | 5659.19 | |
| 672.92 | 499.28 | 30151.71 | |
| 619.75 | 538.51 | 6599.58 | |
| 629.76 | 521.93 | 11627.98 | |
| 165.89 | 295.81 | 16878.70 | |
| 336.86 | 277.27 | 3551.23 | |
| 211.47 | 270.18 | 3446.45 | |
| 321.51 | 430.93 | 11972.92 | |
| 212.89 | 241.62 | 825.55 | |
| 526.45 | 503.20 | 540.47 | |
| 130.54 | 177.12 | 2169.83 | |
| 143.91 | 272.23 | 16464.87 | |
| 314.22 | 479.86 | 27436.00 | |
| 429.73 | 366.64 | 3979.83 | |
| 845.34 | 489.52 | 126607.70 | |
| 204.42 | 297.64 | 8689.36 | |

Average
RMSE: 161.15

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 2.47 | 1.95 | 3.92 | 3.75 | 116.08 |
| 9.02 | 2.31 | 5.47 | 4.44 | 887.32 |
| 1.88 | 3.33 | 2.09 | 3.82 | 85.69 |
| 7.14 | 2.48 | 3.67 | 3.92 | 478.04 |
| 6.23 | 3.15 | 6.72 | 3.41 | 369.31 |
| 8.23 | 2.71 | 5.68 | 4.10 | 688.11 |
| 4.26 | 2.58 | 6.54 | 4.05 | 205.11 |
| 2.27 | 3.46 | 2.07 | 4.31 | 102.08 |
| 8.40 | 2.54 | 5.21 | 3.52 | 736.81 |
| 1.37 | 3.59 | 5.80 | 4.26 | 128.38 |
| 2.20 | 3.63 | 2.06 | 4.20 | 105.80 |
| 8.79 | 3.26 | 6.04 | 3.89 | 818.72 |
| 4.13 | 3.62 | 3.08 | 4.16 | 180.22 |
| 3.14 | 1.77 | 7.10 | 4.20 | 176.23 |
| 1.94 | 2.46 | 3.15 | 3.97 | 87.02 |
| 9.42 | 3.31 | 5.79 | 4.04 | 967.23 |
| 9.18 | 3.04 | 2.75 | 4.23 | 893.78 |
| 2.62 | 2.80 | 4.72 | 4.13 | 113.92 |
| 3.37 | 2.36 | 6.35 | 3.88 | 159.26 |
| 6.92 | 2.31 | 7.01 | 4.22 | 483.61 |
| 4.05 | 2.40 | 6.34 | 3.84 | 192.66 |
| 2.18 | 2.46 | 2.09 | 4.18 | 97.01 |
| 1.53 | 2.72 | 3.60 | 4.22 | 109.24 |
| 2.59 | 2.54 | 2.57 | 4.08 | 107.74 |
| 7.53 | 3.02 | 6.41 | 4.18 | 569.89 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 5.35 | 2.05 | 6.37 | 4.23 | 283.76 |
| 6.00 | 3.25 | 5.47 | 3.85 | 350.77 |
| 6.89 | 2.63 | 6.15 | 3.94 | 459.86 |
| 1.89 | 2.73 | 5.97 | 4.05 | 124.24 |
| 3.38 | 2.66 | 3.89 | 4.01 | 136.89 |
| 8.79 | 3.26 | 6.04 | 3.89 | 818.72 |
| 5.19 | 2.46 | 4.78 | 3.62 | 246.75 |
| 8.09 | 3.55 | 5.27 | 3.79 | 656.89 |
| 5.50 | 2.37 | 3.80 | 4.04 | 289.91 |
| 8.45 | 2.72 | 3.40 | 3.71 | 713.60 |
| 1.53 | 2.72 | 3.60 | 4.22 | 109.24 |
| 9.97 | 2.67 | 3.03 | 3.68 | 1101.66 |
| 1.82 | 2.28 | 3.04 | 4.21 | 101.58 |
| 8.03 | 2.39 | 5.80 | 3.98 | 655.40 |
| 2.20 | 3.63 | 2.06 | 4.20 | 105.80 |
| 2.43 | 2.48 | 7.42 | 3.77 | 147.10 |
| 4.06 | 3.84 | 7.44 | 3.84 | 208.11 |
| 4.20 | 2.94 | 2.72 | 4.22 | 168.33 |
| 6.18 | 3.80 | 3.96 | 3.94 | 335.30 |
| 4.11 | 2.95 | 3.90 | 4.30 | 181.20 |
| 6.48 | 3.75 | 2.15 | 4.09 | 370.25 |
| 1.10 | 3.11 | 2.59 | 4.17 | 103.75 |
| 9.17 | 2.28 | 5.21 | 4.09 | 893.12 |
| 5.89 | 2.80 | 7.61 | 4.02 | 356.75 |
| 2.40 | 2.06 | 2.59 | 4.43 | 126.01 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 6.44 | 2.58 | 7.46 | 3.71 | 413.33 |
| 2.58 | 2.40 | 3.15 | 3.77 | 86.91 |
| 1.05 | 3.42 | 7.46 | 4.23 | 138.90 |
| 2.50 | 2.64 | 6.03 | 4.15 | 140.27 |
| 8.96 | 2.93 | 6.40 | 3.99 | 862.69 |
| 3.79 | 2.29 | 2.32 | 4.11 | 141.22 |
| 1.80 | 3.60 | 7.93 | 4.06 | 161.77 |
| 5.94 | 2.92 | 5.02 | 4.02 | 316.19 |
| 4.17 | 1.75 | 4.63 | 3.78 | 196.33 |
| 6.19 | 3.02 | 4.69 | 3.99 | 357.59 |
| 8.92 | 3.78 | 4.50 | 4.00 | 845.92 |
| 6.94 | 2.99 | 5.65 | 4.13 | 458.89 |
| 1.20 | 2.63 | 2.91 | 3.91 | 99.59 |
| 9.13 | 2.33 | 2.08 | 4.09 | 860.20 |
| 5.89 | 2.80 | 7.61 | 4.02 | 356.75 |
| 8.45 | 2.72 | 3.40 | 3.71 | 713.60 |
| 1.61 | 4.14 | 4.67 | 3.84 | 98.04 |
| 9.46 | 3.59 | 7.02 | 3.80 | 1003.16 |
| 7.97 | 3.02 | 6.57 | 3.99 | 658.66 |
| 6.18 | 3.80 | 3.96 | 3.94 | 335.30 |
| 5.19 | 2.46 | 4.78 | 3.62 | 246.75 |
| 2.56 | 3.17 | 7.54 | 4.15 | 155.10 |
| 3.39 | 3.12 | 5.69 | 3.77 | 165.89 |
| 1.82 | 2.68 | 7.52 | 4.08 | 137.45 |
| 3.38 | 2.66 | 3.89 | 4.01 | 136.89 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 413.33 | 517.98 | 10952.64 | 107.54 |
| 86.91 | 192.00 | 11043.81 | |
| 138.90 | 152.63 | 188.62 | |
| 140.27 | 186.11 | 2101.60 | |
| 862.69 | 698.77 | 26868.66 | |
| 141.22 | 240.15 | 9786.36 | |
| 161.77 | 171.24 | 89.70 | |
| 316.19 | 429.97 | 12945.91 | |
| 196.33 | 296.24 | 9982.94 | |
| 357.59 | 455.98 | 9680.94 | |
| 845.92 | 684.66 | 26003.29 | |
| 458.89 | 524.60 | 4317.41 | |
| 99.59 | 148.60 | 2402.09 | |
| 860.20 | 692.56 | 28104.55 | |
| 356.75 | 432.20 | 5692.63 | |
| 713.60 | 671.18 | 1799.57 | |
| 98.04 | 160.54 | 3906.17 | |
| 1003.16 | 728.35 | 75523.05 | |
| 658.66 | 630.20 | 810.14 | |
| 335.30 | 443.60 | 11728.40 | |
| 246.75 | 391.82 | 21045.87 | |
| 155.10 | 191.43 | 1320.08 | |
| 165.89 | 232.95 | 4497.24 | |
| 137.45 | 170.63 | 1100.59 | |
| 136.89 | 221.79 | 7208.02 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 413.33 | 551.92 | 19206.66 | 147.69 |
| 86.91 | 287.43 | 40208.51 | |
| 138.90 | 157.84 | 358.57 | |
| 140.27 | 201.11 | 3701.69 | |
| 862.69 | 627.31 | 55402.83 | |
| 141.22 | 273.56 | 17513.92 | |
| 161.77 | 178.31 | 273.71 | |
| 316.19 | 392.48 | 5820.32 | |
| 196.33 | 410.27 | 45771.96 | |
| 357.59 | 417.70 | 3613.14 | |
| 845.92 | 583.68 | 68771.93 | |
| 458.89 | 420.68 | 1460.02 | |
| 99.59 | 202.06 | 10499.69 | |
| 860.20 | 656.08 | 41663.38 | |
| 356.75 | 382.87 | 682.50 | |
| 713.60 | 703.73 | 97.45 | |
| 98.04 | 191.99 | 8826.19 | |
| 1003.16 | 684.52 | 101530.33 | |
| 658.66 | 546.89 | 12491.81 | |
| 335.30 | 395.82 | 3662.46 | |
| 246.75 | 508.01 | 68256.19 | |
| 155.10 | 191.53 | 1327.41 | |
| 165.89 | 294.14 | 16447.78 | |
| 137.45 | 189.66 | 2725.68 | |
| 136.89 | 259.37 | 15001.61 | |

Average
RMSE: 127.61

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 4.06 | 3.84 | 7.44 | 3.84 | 208.11 |
| 2.71 | 2.30 | 7.80 | 4.02 | 155.12 |
| 1.64 | 3.36 | 3.18 | 3.98 | 103.04 |
| 8.68 | 2.75 | 6.95 | 3.77 | 794.84 |
| 5.50 | 2.37 | 3.80 | 4.04 | 289.91 |
| 9.13 | 2.33 | 2.08 | 4.09 | 860.20 |
| 7.50 | 2.50 | 7.27 | 3.76 | 567.57 |
| 8.22 | 2.40 | 7.91 | 3.92 | 720.06 |
| 6.22 | 2.14 | 5.46 | 4.10 | 349.88 |
| 6.86 | 2.48 | 7.98 | 3.90 | 485.61 |
| 2.63 | 2.52 | 2.66 | 4.08 | 111.85 |
| 5.93 | 4.10 | 4.77 | 3.75 | 326.03 |
| 6.98 | 2.75 | 5.86 | 4.07 | 479.99 |
| 7.41 | 2.77 | 3.64 | 4.31 | 563.17 |
| 8.23 | 2.71 | 5.68 | 4.10 | 688.11 |
| 3.98 | 2.46 | 4.45 | 3.72 | 151.11 |
| 1.10 | 3.11 | 2.59 | 4.17 | 103.75 |
| 6.49 | 3.15 | 2.72 | 3.68 | 360.99 |
| 6.18 | 2.24 | 5.31 | 3.53 | 349.35 |
| 7.92 | 2.87 | 7.65 | 3.75 | 652.28 |
| 6.38 | 2.42 | 3.72 | 4.43 | 367.83 |
| 5.71 | 2.10 | 3.77 | 4.30 | 300.71 |
| 8.45 | 2.72 | 3.40 | 3.71 | 713.60 |
| 3.82 | 3.55 | 5.03 | 4.23 | 199.07 |
| 2.58 | 2.40 | 3.15 | 3.77 | 86.91 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 6.49 | 3.15 | 2.72 | 3.68 | 360.99 |
| 9.09 | 2.36 | 4.33 | 4.24 | 877.45 |
| 6.25 | 2.47 | 7.18 | 4.00 | 402.72 |
| 1.71 | 3.26 | 3.05 | 3.79 | 83.17 |
| 5.85 | 2.13 | 5.26 | 3.92 | 312.96 |
| 4.74 | 2.77 | 7.84 | 3.81 | 237.71 |
| 5.24 | 2.95 | 6.34 | 3.65 | 272.98 |
| 7.17 | 3.63 | 3.83 | 4.19 | 507.21 |
| 1.54 | 3.12 | 3.70 | 4.00 | 97.47 |
| 5.64 | 3.85 | 4.06 | 3.89 | 295.97 |
| 1.69 | 2.50 | 7.91 | 3.62 | 144.89 |
| 7.02 | 2.87 | 5.05 | 4.12 | 468.41 |
| 4.56 | 3.58 | 3.13 | 3.58 | 196.64 |
| 2.71 | 2.97 | 4.59 | 3.76 | 120.52 |
| 5.62 | 2.13 | 2.21 | 3.95 | 269.32 |
| 7.65 | 2.96 | 4.69 | 4.07 | 550.36 |
| 1.20 | 2.63 | 2.91 | 3.91 | 99.59 |
| 6.05 | 2.43 | 7.73 | 3.89 | 377.36 |
| 9.36 | 2.78 | 2.72 | 3.90 | 923.60 |
| 4.89 | 1.97 | 6.99 | 3.91 | 262.07 |
| 1.02 | 2.28 | 2.60 | 4.15 | 86.61 |
| 9.53 | 3.01 | 3.22 | 3.98 | 988.66 |
| 5.56 | 2.34 | 5.59 | 4.11 | 295.21 |
| 1.08 | 2.38 | 7.05 | 3.95 | 130.26 |
| 8.24 | 3.52 | 4.63 | 3.84 | 694.28 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 7.13 | 2.53 | 7.88 | 3.89 | 517.70 |
| 2.72 | 2.85 | 3.26 | 4.25 | 122.93 |
| 6.92 | 2.31 | 7.01 | 4.22 | 483.61 |
| 8.84 | 3.03 | 4.97 | 4.06 | 817.71 |
| 2.23 | 2.65 | 4.42 | 3.97 | 109.83 |
| 4.61 | 3.25 | 4.73 | 3.86 | 222.29 |
| 1.99 | 2.72 | 2.87 | 3.76 | 101.76 |
| 5.62 | 2.13 | 2.21 | 3.95 | 269.32 |
| 9.40 | 3.26 | 3.11 | 4.22 | 952.67 |
| 4.83 | 2.50 | 4.65 | 4.19 | 232.12 |
| 5.84 | 2.76 | 6.85 | 3.93 | 341.32 |
| 8.39 | 2.22 | 7.84 | 4.14 | 742.67 |
| 9.09 | 3.84 | 2.34 | 3.85 | 865.06 |
| 7.14 | 2.48 | 3.67 | 3.92 | 478.04 |
| 6.19 | 3.02 | 4.69 | 3.99 | 357.59 |
| 8.40 | 2.54 | 5.21 | 3.52 | 736.81 |
| 4.41 | 2.40 | 5.68 | 3.98 | 224.79 |
| 9.04 | 2.76 | 3.89 | 3.61 | 842.89 |
| 7.20 | 3.42 | 5.54 | 4.20 | 526.04 |
| 5.90 | 2.74 | 5.91 | 3.89 | 322.75 |
| 9.26 | 2.25 | 4.46 | 4.32 | 918.17 |
| 2.64 | 2.41 | 5.60 | 3.78 | 141.42 |
| 8.27 | 3.16 | 7.77 | 4.05 | 724.34 |
| 9.24 | 2.36 | 2.25 | 3.93 | 899.95 |
| 4.87 | 2.86 | 2.32 | 3.98 | 202.25 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 517.70 | 224.62 | 85897.80 | 337.07 |
| 122.93 | 210.61 | 7688.38 | |
| 483.61 | 168.36 | 99379.48 | |
| 817.71 | 614.85 | 41151.08 | |
| 109.83 | 347.15 | 56322.99 | |
| 222.29 | 611.97 | 151849.72 | |
| 101.76 | 562.72 | 212487.81 | |
| 269.32 | 600.69 | 109804.09 | |
| 952.67 | 434.07 | 268945.65 | |
| 232.12 | 508.78 | 76538.93 | |
| 341.32 | 189.72 | 22982.44 | |
| 742.67 | 310.83 | 186488.23 | |
| 865.06 | 466.30 | 159006.60 | |
| 478.04 | 457.39 | 426.60 | |
| 357.59 | 562.97 | 42182.16 | |
| 736.81 | 265.71 | 221934.96 | |
| 224.79 | 162.98 | 3820.51 | |
| 842.89 | 413.15 | 184676.91 | |
| 526.04 | 486.62 | 1553.61 | |
| 322.75 | 574.46 | 63358.61 | |
| 918.17 | 375.76 | 294211.36 | |
| 141.42 | 354.32 | 45325.99 | |
| 724.34 | 592.49 | 17383.90 | |
| 899.95 | 202.15 | 486918.14 | |
| 202.25 | 200.35 | 3.60 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 517.70 | 394.51 | 15176.11 | 145.50 |
| 122.93 | 248.46 | 15757.53 | |
| 483.61 | 459.13 | 599.48 | |
| 817.71 | 639.68 | 31695.36 | |
| 109.83 | 209.80 | 9994.03 | |
| 222.29 | 273.84 | 2657.53 | |
| 101.76 | 208.48 | 11388.55 | |
| 269.32 | 488.67 | 48116.18 | |
| 952.67 | 709.43 | 59165.49 | |
| 232.12 | 346.08 | 12985.89 | |
| 341.32 | 320.76 | 422.72 | |
| 742.67 | 558.64 | 33867.79 | |
| 865.06 | 669.90 | 38088.20 | |
| 478.04 | 554.94 | 5912.89 | |
| 357.59 | 412.28 | 2991.41 | |
| 736.81 | 559.33 | 31500.66 | |
| 224.79 | 276.64 | 2688.81 | |
| 842.89 | 646.54 | 38552.67 | |
| 526.04 | 483.70 | 1792.56 | |
| 322.75 | 347.99 | 637.27 | |
| 918.17 | 711.45 | 42732.59 | |
| 141.42 | 206.40 | 4223.01 | |
| 724.34 | 494.10 | 53011.77 | |
| 899.95 | 717.58 | 33257.17 | |
| 202.25 | 381.23 | 32032.61 | |

Average
RMSE 241.28

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.01 | 3.04 | 7.51 | 4.17 | 686.90 |
| 3.81 | 2.39 | 6.20 | 3.68 | 185.03 |
| 2.37 | 2.93 | 7.66 | 3.96 | 155.45 |
| 2.70 | 3.47 | 6.47 | 4.19 | 171.87 |
| 8.17 | 3.27 | 4.85 | 4.06 | 702.34 |
| 2.77 | 2.52 | 2.47 | 4.00 | 113.58 |
| 4.52 | 3.28 | 4.83 | 4.19 | 218.05 |
| 8.13 | 2.95 | 5.61 | 3.87 | 676.06 |
| 7.60 | 3.21 | 5.00 | 4.10 | 585.09 |
| 3.64 | 3.30 | 3.46 | 4.08 | 171.26 |
| 4.87 | 2.61 | 4.51 | 3.65 | 219.97 |
| 1.56 | 3.42 | 6.67 | 4.17 | 107.76 |
| 1.02 | 2.28 | 2.60 | 4.15 | 86.61 |
| 6.69 | 2.18 | 4.44 | 3.95 | 394.49 |
| 3.93 | 2.58 | 3.12 | 3.79 | 173.29 |
| 7.32 | 3.18 | 3.53 | 3.78 | 514.00 |
| 4.11 | 2.95 | 3.90 | 4.30 | 181.20 |
| 1.64 | 2.59 | 5.65 | 3.96 | 137.57 |
| 9.42 | 3.31 | 5.79 | 4.04 | 967.23 |
| 1.71 | 3.26 | 3.05 | 3.79 | 83.17 |
| 5.16 | 2.36 | 3.34 | 4.18 | 242.60 |
| 8.45 | 2.72 | 3.40 | 3.71 | 713.60 |
| 5.99 | 1.96 | 6.13 | 3.66 | 326.85 |
| 2.38 | 3.10 | 7.54 | 4.04 | 168.96 |
| 1.05 | 3.42 | 7.46 | 4.23 | 138.90 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 6.44 | 2.58 | 7.46 | 3.71 | 413.33 |
| 9.36 | 2.78 | 2.72 | 3.90 | 923.60 |
| 5.00 | 2.42 | 5.69 | 3.80 | 245.03 |
| 5.16 | 2.36 | 3.34 | 4.18 | 242.60 |
| 3.79 | 2.29 | 2.32 | 4.11 | 141.22 |
| 6.33 | 2.00 | 4.49 | 3.71 | 360.77 |
| 4.53 | 2.69 | 2.73 | 4.17 | 189.20 |
| 3.72 | 2.77 | 2.05 | 3.92 | 133.14 |
| 8.77 | 4.23 | 4.43 | 4.30 | 819.25 |
| 4.39 | 3.24 | 4.71 | 4.01 | 217.17 |
| 2.45 | 3.32 | 4.65 | 4.00 | 120.87 |
| 7.64 | 2.57 | 5.99 | 4.33 | 569.96 |
| 3.80 | 3.05 | 2.47 | 3.97 | 166.03 |
| 2.18 | 2.46 | 2.09 | 4.18 | 97.01 |
| 7.45 | 2.70 | 4.22 | 4.18 | 536.13 |
| 1.12 | 2.98 | 5.69 | 4.22 | 119.86 |
| 7.92 | 2.24 | 3.47 | 3.65 | 602.92 |
| 5.85 | 2.62 | 3.62 | 3.88 | 328.05 |
| 1.99 | 2.72 | 2.87 | 3.76 | 101.76 |
| 6.35 | 2.25 | 4.61 | 4.38 | 392.80 |
| 7.13 | 3.26 | 5.23 | 4.11 | 494.21 |
| 3.64 | 3.14 | 4.96 | 4.02 | 164.56 |
| 2.47 | 1.95 | 3.92 | 3.75 | 116.08 |
| 9.94 | 2.85 | 3.83 | 4.03 | 1107.42 |
| 8.12 | 3.12 | 2.24 | 4.10 | 646.90 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 7.35 | 2.76 | 4.69 | 3.95 | 510.36 |
| 3.38 | 3.44 | 3.07 | 3.63 | 123.79 |
| 2.80 | 2.61 | 4.25 | 3.88 | 115.62 |
| 1.77 | 2.45 | 2.18 | 3.91 | 76.79 |
| 5.81 | 2.42 | 4.31 | 3.63 | 321.51 |
| 1.00 | 2.16 | 3.47 | 4.39 | 106.17 |
| 9.40 | 3.26 | 3.11 | 4.22 | 952.67 |
| 8.79 | 2.34 | 2.60 | 3.94 | 775.37 |
| 1.38 | 2.35 | 2.05 | 3.99 | 90.93 |
| 2.38 | 2.65 | 5.00 | 3.63 | 127.63 |
| 1.08 | 2.38 | 7.05 | 3.95 | 130.26 |
| 4.69 | 3.32 | 5.68 | 3.99 | 227.61 |
| 4.08 | 2.88 | 2.43 | 4.13 | 162.23 |
| 8.14 | 2.86 | 4.94 | 4.19 | 674.04 |
| 1.02 | 2.28 | 2.60 | 4.15 | 86.61 |
| 9.34 | 2.63 | 6.61 | 4.01 | 961.15 |
| 1.05 | 3.42 | 7.46 | 4.23 | 138.90 |
| 2.58 | 3.73 | 4.04 | 3.80 | 132.49 |
| 5.30 | 2.74 | 5.91 | 3.89 | 322.75 |
| 9.81 | 3.18 | 2.62 | 3.70 | 1050.38 |
| 5.46 | 2.83 | 4.06 | 3.82 | 277.11 |
| 1.23 | 2.93 | 3.59 | 4.04 | 92.39 |
| 5.99 | 1.96 | 6.13 | 3.66 | 326.85 |
| 1.26 | 2.08 | 6.06 | 3.81 | 127.72 |
| 7.07 | 2.70 | 5.37 | 4.05 | 488.96 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 510.36 | 510.38 | 0.00 | 149.93 |
| 123.79 | 288.42 | 27103.29 | |
| 115.62 | 212.91 | 9464.82 | |
| 76.79 | 177.62 | 10167.25 | |
| 321.51 | 417.44 | 9202.43 | |
| 106.17 | 152.91 | 2184.42 | |
| 952.67 | 625.27 | 107189.50 | |
| 775.37 | 585.12 | 36193.72 | |
| 90.93 | 166.27 | 5675.74 | |
| 127.63 | 214.17 | 7489.51 | |
| 130.26 | 164.47 | 1170.04 | |
| 227.61 | 329.57 | 10396.12 | |
| 162.23 | 257.06 | 8992.07 | |
| 674.04 | 540.92 | 17722.01 | |
| 86.61 | 157.09 | 4967.59 | |
| 961.15 | 622.01 | 115019.18 | |
| 138.90 | 168.71 | 888.88 | |
| 132.49 | 239.88 | 11533.66 | |
| 322.75 | 406.15 | 6956.27 | |
| 1050.38 | 670.61 | 144221.74 | |
| 277.11 | 384.49 | 11530.53 | |
| 92.39 | 169.63 | 5966.20 | |
| 326.85 | 405.01 | 6108.75 | |
| 127.72 | 167.15 | 1555.05 | |
| 488.96 | 473.27 | 246.27 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 510.36 | 522.24 | 141.25 | 134.55 |
| 123.79 | 262.35 | 19197.71 | |
| 115.62 | 213.84 | 9646.84 | |
| 76.79 | 186.22 | 11974.22 | |
| 321.51 | 395.01 | 5402.47 | |
| 106.17 | 164.09 | 3355.11 | |
| 952.67 | 674.16 | 77565.58 | |
| 775.37 | 625.97 | 22319.75 | |
| 90.93 | 176.46 | 7315.51 | |
| 127.63 | 203.09 | 5694.23 | |
| 130.26 | 166.11 | 1285.27 | |
| 227.61 | 317.69 | 8115.21 | |
| 162.23 | 276.85 | 13138.47 | |
| 674.04 | 576.69 | 9476.93 | |
| 86.61 | 167.70 | 6575.32 | |
| 961.15 | 649.19 | 97317.80 | |
| 138.90 | 170.25 | 982.89 | |
| 132.49 | 224.84 | 8528.53 | |
| 322.75 | 394.22 | 5107.54 | |
| 1050.38 | 706.53 | 118231.97 | |
| 277.11 | 374.21 | 9427.78 | |
| 92.39 | 175.83 | 6961.52 | |
| 326.85 | 382.90 | 3141.78 | |
| 127.72 | 168.99 | 1703.03 | |
| 488.96 | 488.44 | 0.27 | |

| Average | |
|---|---|
| RMSE | 142.24 |

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.72 | 3.02 | 7.11 | 4.02 | 137.50 |
| 9.98 | 2.42 | 2.72 | 4.24 | 1109.20 |
| 5.76 | 3.10 | 7.63 | 3.67 | 349.75 |
| 7.02 | 2.62 | 7.56 | 4.09 | 488.64 |
| 8.03 | 2.39 | 5.80 | 3.98 | 655.40 |
| 6.53 | 2.94 | 7.44 | 3.79 | 419.52 |
| 8.65 | 2.76 | 5.24 | 4.19 | 793.63 |
| 9.34 | 2.63 | 6.61 | 4.01 | 961.15 |
| 6.31 | 3.43 | 3.22 | 4.04 | 361.47 |
| 4.63 | 2.45 | 3.71 | 4.20 | 225.29 |
| 7.60 | 3.21 | 5.00 | 4.10 | 585.09 |
| 4.16 | 2.39 | 2.85 | 4.13 | 169.88 |
| 6.92 | 2.31 | 7.01 | 4.22 | 483.61 |
| 9.97 | 2.67 | 3.03 | 3.68 | 1101.66 |
| 2.64 | 2.41 | 5.60 | 3.78 | 141.42 |
| 5.91 | 2.78 | 4.61 | 3.90 | 347.85 |
| 9.44 | 2.98 | 5.74 | 3.89 | 979.19 |
| 5.13 | 2.97 | 2.25 | 3.97 | 232.56 |
| 4.09 | 3.48 | 6.12 | 3.69 | 185.78 |
| 4.39 | 3.24 | 4.71 | 4.01 | 217.17 |
| 5.64 | 2.04 | 6.58 | 4.11 | 338.63 |
| 5.24 | 2.95 | 6.34 | 3.65 | 272.98 |
| 2.72 | 2.31 | 7.64 | 3.64 | 154.07 |
| 9.17 | 2.28 | 5.21 | 4.09 | 893.12 |
| 6.33 | 2.00 | 4.49 | 3.71 | 360.77 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.77 | 2.32 | 6.92 | 3.76 | 812.19 |
| 3.29 | 3.10 | 4.34 | 3.81 | 149.75 |
| 8.09 | 3.55 | 5.27 | 3.79 | 656.89 |
| 1.03 | 4.30 | 5.87 | 3.98 | 97.50 |
| 9.24 | 2.36 | 2.25 | 3.93 | 899.95 |
| 7.13 | 2.53 | 7.88 | 3.89 | 517.70 |
| 8.79 | 3.26 | 6.04 | 3.89 | 818.72 |
| 5.90 | 1.93 | 6.22 | 3.91 | 343.19 |
| 2.81 | 3.00 | 2.76 | 3.85 | 133.15 |
| 1.35 | 2.50 | 4.74 | 3.91 | 102.81 |
| 1.25 | 3.32 | 2.40 | 4.15 | 111.27 |
| 6.84 | 2.61 | 4.66 | 4.09 | 427.61 |
| 6.18 | 2.24 | 5.31 | 3.53 | 349.35 |
| 1.64 | 2.59 | 5.65 | 3.96 | 137.57 |
| 6.81 | 2.36 | 2.02 | 3.98 | 429.73 |
| 2.36 | 2.62 | 2.93 | 4.16 | 130.54 |
| 7.30 | 1.78 | 3.01 | 3.95 | 489.89 |
| 5.85 | 3.31 | 3.00 | 4.02 | 311.22 |
| 5.62 | 2.13 | 2.21 | 3.95 | 269.32 |
| 4.51 | 3.24 | 4.58 | 4.37 | 214.65 |
| 2.71 | 2.56 | 3.29 | 3.89 | 116.91 |
| 7.45 | 2.70 | 4.22 | 4.18 | 536.13 |
| 6.24 | 2.72 | 3.59 | 3.87 | 337.23 |
| 8.96 | 2.93 | 6.40 | 3.99 | 862.69 |
| 7.64 | 2.57 | 5.99 | 4.33 | 569.96 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 7.30 | 1.78 | 3.01 | 3.95 | 489.89 |
| 8.62 | 2.39 | 3.45 | 4.51 | 760.18 |
| 2.38 | 2.80 | 6.35 | 3.77 | 138.34 |
| 2.80 | 2.61 | 4.25 | 3.88 | 115.62 |
| 6.01 | 1.91 | 2.78 | 3.64 | 315.36 |
| 9.54 | 2.54 | 2.23 | 4.02 | 983.71 |
| 2.81 | 2.05 | 7.65 | 3.96 | 174.24 |
| 6.92 | 2.99 | 5.15 | 4.04 | 466.84 |
| 9.81 | 3.18 | 2.62 | 3.70 | 1050.38 |
| 9.19 | 2.09 | 3.92 | 3.71 | 896.59 |
| 9.13 | 2.33 | 2.08 | 4.09 | 860.20 |
| 2.72 | 2.85 | 3.26 | 4.25 | 122.93 |
| 2.43 | 2.48 | 7.42 | 3.77 | 147.10 |
| 6.88 | 3.69 | 6.42 | 4.11 | 499.42 |
| 5.32 | 2.94 | 4.26 | 4.11 | 252.90 |
| 4.06 | 3.84 | 7.44 | 3.84 | 208.11 |
| 4.44 | 2.81 | 7.31 | 4.34 | 238.50 |
| 2.53 | 2.84 | 3.62 | 3.86 | 118.78 |
| 7.02 | 2.62 | 7.56 | 4.09 | 488.64 |
| 3.54 | 2.41 | 5.33 | 3.86 | 175.59 |
| 3.07 | 3.32 | 7.01 | 3.99 | 167.51 |
| 8.21 | 3.37 | 3.97 | 4.37 | 681.74 |
| 3.64 | 3.30 | 3.46 | 4.08 | 171.26 |
| 9.16 | 2.58 | 6.61 | 4.13 | 905.39 |
| 4.87 | 2.86 | 2.32 | 3.98 | 202.25 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 489.89 | 703.90 | 45799.50 | 103.78 |
| 760.18 | 753.44 | 45.44 | |
| 138.34 | 188.45 | 2511.18 | |
| 115.62 | 211.72 | 9235.74 | |
| 315.36 | 552.53 | 56249.49 | |
| 983.71 | 858.17 | 15759.24 | |
| 174.24 | 207.07 | 1077.69 | |
| 466.84 | 474.96 | 65.91 | |
| 1050.38 | 839.64 | 44413.04 | |
| 896.59 | 847.03 | 2455.81 | |
| 860.20 | 841.43 | 352.48 | |
| 122.93 | 204.64 | 6676.91 | |
| 147.10 | 191.03 | 1929.46 | |
| 499.42 | 373.40 | 15882.16 | |
| 252.90 | 328.27 | 5681.14 | |
| 208.11 | 201.96 | 37.77 | |
| 238.50 | 240.41 | 3.67 | |
| 118.78 | 201.89 | 6906.86 | |
| 488.64 | 473.27 | 236.25 | |
| 175.59 | 239.42 | 4074.54 | |
| 167.51 | 189.59 | 487.72 | |
| 681.74 | 600.28 | 6636.27 | |
| 171.26 | 222.59 | 2635.10 | |
| 905.39 | 751.73 | 23610.02 | |
| 202.25 | 330.74 | 16510.83 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 489.89 | 546.84 | 3242.82 | 74.33 |
| 760.18 | 698.77 | 3771.06 | |
| 138.34 | 143.90 | 30.93 | |
| 115.62 | 150.51 | 1217.07 | |
| 315.36 | 365.99 | 2563.38 | |
| 983.71 | 810.48 | 30008.82 | |
| 174.24 | 151.83 | 502.02 | |
| 466.84 | 453.75 | 171.40 | |
| 1050.38 | 831.92 | 47726.09 | |
| 896.59 | 793.88 | 10548.63 | |
| 860.20 | 772.63 | 7668.30 | |
| 122.93 | 146.77 | 568.17 | |
| 147.10 | 145.27 | 3.34 | |
| 499.42 | 426.54 | 5312.19 | |
| 252.90 | 256.89 | 15.92 | |
| 208.11 | 179.66 | 809.68 | |
| 238.50 | 196.77 | 1741.74 | |
| 118.78 | 145.68 | 723.54 | |
| 488.64 | 479.01 | 92.82 | |
| 175.59 | 169.13 | 41.71 | |
| 167.51 | 153.08 | 208.31 | |
| 681.74 | 624.48 | 3278.55 | |
| 171.26 | 166.09 | 26.73 | |
| 905.39 | 774.11 | 17234.83 | |
| 202.25 | 226.88 | 606.72 | |

Average
RMSE 89.06

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.70 | 2.86 | 4.19 | 4.13 | 91.37 |
| 4.85 | 2.70 | 2.50 | 4.09 | 217.86 |
| 5.85 | 2.13 | 5.26 | 3.92 | 312.96 |
| 8.57 | 2.74 | 4.38 | 3.87 | 750.31 |
| 9.80 | 2.22 | 2.79 | 4.27 | 1070.64 |
| 1.36 | 2.85 | 7.31 | 3.92 | 141.27 |
| 7.17 | 2.93 | 5.61 | 4.04 | 505.13 |
| 7.01 | 2.81 | 5.07 | 3.91 | 468.90 |
| 2.62 | 2.80 | 4.72 | 4.13 | 113.92 |
| 1.34 | 1.95 | 6.24 | 4.45 | 117.52 |
| 1.64 | 3.36 | 3.18 | 3.98 | 103.04 |
| 1.94 | 2.46 | 3.15 | 3.97 | 87.02 |
| 6.09 | 2.80 | 2.54 | 3.94 | 324.80 |
| 4.87 | 2.61 | 4.51 | 3.65 | 219.97 |
| 8.23 | 2.71 | 5.68 | 4.10 | 688.11 |
| 9.73 | 3.09 | 7.87 | 3.78 | 1080.12 |
| 1.10 | 2.66 | 2.30 | 3.80 | 97.42 |
| 7.80 | 3.20 | 7.46 | 3.92 | 610.80 |
| 8.27 | 3.16 | 7.77 | 4.05 | 724.34 |
| 5.42 | 2.91 | 2.12 | 4.17 | 263.19 |
| 7.64 | 2.57 | 5.99 | 4.33 | 569.96 |
| 2.03 | 2.41 | 6.88 | 3.80 | 128.79 |
| 6.81 | 2.36 | 2.02 | 3.98 | 429.73 |
| 6.86 | 2.48 | 7.98 | 3.90 | 485.61 |
| 8.12 | 3.05 | 7.96 | 4.07 | 714.80 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 4.44 | 2.81 | 7.31 | 4.34 | 238.50 |
| 8.27 | 3.16 | 7.77 | 4.05 | 724.34 |
| 5.89 | 2.80 | 7.61 | 4.02 | 356.75 |
| 7.50 | 2.50 | 7.27 | 3.76 | 567.57 |
| 4.64 | 2.85 | 7.54 | 3.89 | 219.22 |
| 8.21 | 3.37 | 3.97 | 4.37 | 681.74 |
| 6.77 | 3.16 | 5.13 | 4.05 | 447.51 |
| 2.58 | 2.40 | 3.15 | 3.77 | 86.91 |
| 5.00 | 2.42 | 5.69 | 3.80 | 245.03 |
| 4.13 | 2.76 | 6.72 | 3.95 | 187.61 |
| 3.01 | 2.61 | 2.93 | 4.15 | 126.62 |
| 4.52 | 3.28 | 4.83 | 4.19 | 218.05 |
| 5.86 | 2.86 | 4.45 | 3.80 | 311.17 |
| 9.94 | 2.85 | 3.83 | 4.03 | 1107.42 |
| 7.41 | 2.77 | 3.64 | 4.31 | 563.17 |
| 5.98 | 2.93 | 2.55 | 4.06 | 314.65 |
| 6.43 | 3.24 | 6.32 | 4.16 | 392.85 |
| 6.68 | 2.84 | 7.80 | 4.07 | 470.40 |
| 7.02 | 2.87 | 5.05 | 4.12 | 468.41 |
| 5.64 | 2.04 | 6.58 | 4.11 | 338.63 |
| 8.73 | 2.09 | 5.30 | 4.19 | 772.98 |
| 3.24 | 2.70 | 3.29 | 3.89 | 120.70 |
| 3.38 | 3.44 | 3.07 | 3.63 | 123.79 |
| 3.54 | 2.41 | 5.33 | 3.86 | 175.59 |
| 3.91 | 3.48 | 5.76 | 3.56 | 172.30 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 7.14 | 2.48 | 3.67 | 3.92 | 478.04 |
| 1.00 | 2.16 | 3.47 | 4.39 | 106.17 |
| 3.38 | 2.66 | 3.89 | 4.01 | 136.89 |
| 6.00 | 3.25 | 5.47 | 3.85 | 350.77 |
| 9.53 | 3.01 | 3.22 | 3.98 | 988.66 |
| 5.13 | 2.44 | 3.46 | 3.84 | 246.42 |
| 3.79 | 2.29 | 2.32 | 4.11 | 141.22 |
| 7.75 | 2.32 | 7.12 | 4.21 | 604.77 |
| 5.13 | 2.97 | 2.25 | 3.97 | 232.56 |
| 6.24 | 2.50 | 7.89 | 4.16 | 413.48 |
| 5.95 | 2.61 | 3.14 | 3.89 | 304.78 |
| 1.92 | 3.59 | 3.46 | 4.04 | 102.97 |
| 2.51 | 3.25 | 3.76 | 4.01 | 140.73 |
| 8.65 | 2.76 | 5.24 | 4.19 | 793.63 |
| 3.30 | 2.70 | 2.11 | 4.21 | 132.71 |
| 2.39 | 2.21 | 5.46 | 3.60 | 121.95 |
| 2.01 | 2.30 | 3.95 | 4.09 | 106.91 |
| 8.02 | 2.54 | 5.01 | 3.85 | 640.83 |
| 6.49 | 3.15 | 2.72 | 3.68 | 360.99 |
| 8.59 | 2.94 | 4.56 | 3.94 | 771.73 |
| 2.72 | 2.85 | 3.26 | 4.25 | 122.93 |
| 2.45 | 3.32 | 4.65 | 4.00 | 120.87 |
| 1.43 | 1.45 | 3.72 | 4.13 | 111.58 |
| 6.36 | 3.12 | 7.46 | 3.98 | 405.95 |
| 6.92 | 2.99 | 5.15 | 4.04 | 466.84 |

**Model a**

| *Desired* | *Actual* | SE | RMSE |
|---|---|---|---|
| 478.04 | 486.38 | 69.61 | 54.71 |
| 106.17 | 112.99 | 46.51 | |
| 136.89 | 135.43 | 2.12 | |
| 350.77 | 261.91 | 7895.76 | |
| 988.66 | 927.23 | 3774.15 | |
| 246.42 | 225.04 | 457.12 | |
| 141.22 | 144.55 | 11.09 | |
| 604.77 | 609.57 | 23.04 | |
| 232.56 | 213.19 | 375.22 | |
| 413.48 | 249.76 | 26804.09 | |
| 304.78 | 296.53 | 68.11 | |
| 102.97 | 126.91 | 573.26 | |
| 140.73 | 129.02 | 137.16 | |
| 793.63 | 811.93 | 334.82 | |
| 132.71 | 133.59 | 0.77 | |
| 121.95 | 129.76 | 61.01 | |
| 106.91 | 118.86 | 142.81 | |
| 640.83 | 717.36 | 5857.14 | |
| 360.99 | 406.90 | 2108.10 | |
| 771.73 | 808.63 | 1361.37 | |
| 122.93 | 125.14 | 4.89 | |
| 120.87 | 127.41 | 42.83 | |
| 111.58 | 113.19 | 2.60 | |
| 405.95 | 275.23 | 17087.50 | |
| 466.84 | 379.76 | 7582.58 | |

**Model b**

| *Desired* | *Actual* | SE | RMSE |
|---|---|---|---|
| 478.04 | 493.94 | 252.88 | 47.70 |
| 106.17 | 131.30 | 631.46 | |
| 136.89 | 140.69 | 14.44 | |
| 350.77 | 270.30 | 6475.14 | |
| 988.66 | 910.75 | 6069.90 | |
| 246.42 | 201.77 | 1993.86 | |
| 141.22 | 149.17 | 63.16 | |
| 604.77 | 539.36 | 4278.90 | |
| 232.56 | 185.97 | 2171.02 | |
| 413.48 | 323.95 | 8015.39 | |
| 304.78 | 261.98 | 1831.51 | |
| 102.97 | 129.75 | 717.12 | |
| 140.73 | 132.03 | 75.69 | |
| 793.63 | 771.52 | 488.66 | |
| 132.71 | 142.31 | 92.11 | |
| 121.95 | 162.00 | 1603.97 | |
| 106.91 | 132.84 | 672.38 | |
| 640.83 | 699.35 | 3424.22 | |
| 360.99 | 410.96 | 2496.73 | |
| 771.73 | 830.26 | 3426.22 | |
| 122.93 | 136.20 | 176.13 | |
| 120.87 | 131.62 | 115.46 | |
| 111.58 | 133.88 | 497.43 | |
| 405.95 | 321.64 | 7108.43 | |
| 466.84 | 402.05 | 4197.88 | |

**Average**
**RMSE:** 51.21

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 3.98 | 2.46 | 4.45 | 3.72 | 151.11 |
| 7.63 | 2.70 | 3.42 | 4.10 | 564.34 |
| 7.80 | 3.30 | 3.44 | 3.82 | 567.99 |
| 5.69 | 2.05 | 2.27 | 3.94 | 271.61 |
| 6.69 | 2.46 | 3.20 | 4.13 | 424.55 |
| 1.25 | 3.32 | 2.40 | 4.15 | 111.27 |
| 7.50 | 2.50 | 7.27 | 3.76 | 567.57 |
| 8.45 | 2.72 | 3.40 | 3.71 | 713.60 |
| 3.81 | 2.39 | 6.20 | 3.68 | 185.03 |
| 5.94 | 2.92 | 5.02 | 4.02 | 316.19 |
| 8.14 | 2.86 | 4.94 | 4.19 | 674.04 |
| 1.88 | 2.97 | 3.54 | 4.10 | 98.27 |
| 1.00 | 2.16 | 3.47 | 4.39 | 106.17 |
| 4.33 | 2.77 | 7.60 | 3.93 | 228.76 |
| 4.54 | 3.42 | 4.22 | 4.14 | 207.60 |
| 1.01 | 2.43 | 6.08 | 4.18 | 133.72 |
| 5.32 | 2.94 | 4.26 | 4.11 | 252.90 |
| 8.59 | 2.94 | 4.56 | 3.94 | 771.73 |
| 8.24 | 3.52 | 4.63 | 3.84 | 694.28 |
| 8.57 | 2.74 | 4.38 | 3.87 | 750.31 |
| 4.74 | 2.77 | 7.84 | 3.81 | 237.71 |
| 9.34 | 2.30 | 2.08 | 3.82 | 905.66 |
| 7.75 | 2.32 | 7.12 | 4.21 | 604.77 |
| 9.46 | 3.59 | 7.02 | 3.80 | 1003.16 |
| 4.40 | 3.31 | 6.71 | 4.16 | 219.63 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 5.69 | 2.05 | 2.27 | 3.94 | 271.61 |
| 5.71 | 2.10 | 3.77 | 4.30 | 300.71 |
| 2.59 | 2.54 | 2.57 | 4.08 | 107.74 |
| 7.21 | 3.20 | 3.49 | 3.95 | 489.68 |
| 1.61 | 4.14 | 4.67 | 3.84 | 98.04 |
| 6.69 | 2.18 | 4.44 | 3.95 | 394.49 |
| 2.50 | 2.75 | 3.60 | 4.25 | 118.98 |
| 9.94 | 3.42 | 2.39 | 3.99 | 1090.88 |
| 4.14 | 3.24 | 6.01 | 4.00 | 204.42 |
| 9.65 | 2.82 | 4.57 | 3.89 | 1006.50 |
| 2.26 | 3.22 | 2.91 | 4.06 | 97.67 |
| 3.79 | 2.29 | 2.32 | 4.11 | 141.22 |
| 1.89 | 2.73 | 5.97 | 4.05 | 124.24 |
| 7.02 | 2.62 | 7.56 | 4.09 | 488.64 |
| 4.41 | 2.40 | 5.68 | 3.96 | 224.79 |
| 7.41 | 2.27 | 5.85 | 3.97 | 538.15 |
| 1.88 | 2.97 | 3.54 | 4.10 | 98.27 |
| 8.18 | 2.85 | 6.03 | 4.25 | 694.60 |
| 6.89 | 2.63 | 6.15 | 3.94 | 459.86 |
| 8.86 | 2.95 | 4.85 | 4.04 | 821.76 |
| 5.34 | 2.61 | 2.83 | 4.09 | 256.90 |
| 2.09 | 3.12 | 3.66 | 4.19 | 121.92 |
| 5.51 | 3.00 | 2.38 | 4.08 | 281.92 |
| 2.72 | 2.85 | 3.26 | 4.25 | 122.93 |
| 9.54 | 2.54 | 2.23 | 4.02 | 983.71 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.81 | 2.45 | 5.29 | 3.99 | 104.23 |
| 2.58 | 3.73 | 4.04 | 3.80 | 132.49 |
| 5.90 | 2.74 | 5.91 | 3.89 | 322.75 |
| 4.93 | 3.19 | 7.27 | 4.07 | 275.68 |
| 3.47 | 2.21 | 7.44 | 4.13 | 175.75 |
| 8.86 | 2.95 | 4.85 | 4.04 | 821.76 |
| 4.14 | 3.24 | 6.01 | 4.00 | 204.42 |
| 2.45 | 3.32 | 4.65 | 4.00 | 120.87 |
| 1.53 | 3.19 | 6.28 | 4.03 | 129.40 |
| 6.34 | 2.69 | 6.51 | 4.05 | 400.19 |
| 2.53 | 2.84 | 3.62 | 3.86 | 118.78 |
| 2.36 | 2.62 | 2.93 | 4.16 | 130.54 |
| 2.77 | 2.52 | 2.47 | 4.00 | 113.58 |
| 5.64 | 2.04 | 6.58 | 4.11 | 338.63 |
| 8.39 | 2.22 | 7.84 | 4.14 | 742.67 |
| 7.60 | 3.21 | 5.00 | 4.10 | 585.09 |
| 7.64 | 2.57 | 5.99 | 4.33 | 569.96 |
| 4.74 | 2.77 | 7.84 | 3.81 | 237.71 |
| 4.20 | 2.94 | 2.72 | 4.22 | 168.33 |
| 1.03 | 2.95 | 6.41 | 3.96 | 122.22 |
| 8.51 | 2.60 | 6.10 | 3.86 | 735.82 |
| 3.87 | 3.68 | 2.16 | 3.87 | 151.90 |
| 5.00 | 2.42 | 5.69 | 3.80 | 245.03 |
| 7.19 | 3.42 | 2.06 | 3.85 | 469.72 |
| 3.30 | 2.70 | 2.11 | 4.21 | 132.71 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 104.23 | 132.22 | 783.28 | 30.42 |
| 132.49 | 147.26 | 218.15 | |
| 322.75 | 339.75 | 289.07 | |
| 275.68 | 231.71 | 1933.62 | |
| 175.75 | 158.75 | 288.88 | |
| 821.76 | 818.55 | 10.31 | |
| 204.42 | 186.46 | 322.40 | |
| 120.87 | 143.44 | 509.61 | |
| 129.40 | 131.34 | 3.75 | |
| 400.19 | 399.42 | 0.59 | |
| 118.78 | 143.05 | 589.20 | |
| 130.54 | 142.01 | 131.49 | |
| 113.58 | 147.46 | 1148.11 | |
| 338.63 | 297.14 | 1721.06 | |
| 742.67 | 771.94 | 856.51 | |
| 585.09 | 610.82 | 661.88 | |
| 569.96 | 607.63 | 1418.72 | |
| 237.71 | 222.54 | 230.26 | |
| 168.33 | 191.29 | 527.27 | |
| 122.22 | 126.71 | 20.19 | |
| 735.82 | 802.95 | 4507.01 | |
| 151.90 | 184.41 | 1056.77 | |
| 245.03 | 241.40 | 13.14 | |
| 469.72 | 540.79 | 5050.28 | |
| 132.71 | 161.67 | 838.50 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 104.23 | 123.24 | 361.26 | 42.80 |
| 132.49 | 159.95 | 754.22 | |
| 322.75 | 345.98 | 539.58 | |
| 275.68 | 301.31 | 657.02 | |
| 175.75 | 184.62 | 78.63 | |
| 821.76 | 879.63 | 3348.59 | |
| 204.42 | 187.79 | 276.51 | |
| 120.87 | 120.88 | 0.00 | |
| 129.40 | 111.21 | 330.79 | |
| 400.19 | 462.45 | 3876.27 | |
| 118.78 | 156.98 | 1459.16 | |
| 130.54 | 109.15 | 457.59 | |
| 113.58 | 134.68 | 445.37 | |
| 338.63 | 393.64 | 3026.44 | |
| 742.67 | 676.66 | 4357.64 | |
| 585.09 | 563.46 | 468.04 | |
| 569.96 | 561.73 | 67.67 | |
| 237.71 | 381.34 | 20628.64 | |
| 168.33 | 126.93 | 1713.67 | |
| 122.22 | 114.49 | 59.77 | |
| 735.82 | 706.98 | 832.84 | |
| 151.90 | 182.35 | 927.40 | |
| 245.03 | 256.62 | 134.39 | |
| 469.72 | 443.03 | 712.38 | |
| 132.71 | 115.64 | 291.36 | |

Average
RMSE: 36.61

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 5.90 | 2.74 | 5.91 | 3.89 | 322.75 |
| 5.59 | 2.05 | 5.69 | 4.33 | 304.20 |
| 3.82 | 3.55 | 5.03 | 4.23 | 199.07 |
| 9.73 | 3.09 | 7.87 | 3.78 | 1080.12 |
| 8.24 | 3.52 | 4.63 | 3.84 | 694.28 |
| 3.42 | 2.56 | 5.49 | 3.95 | 168.57 |
| 5.53 | 2.73 | 7.84 | 3.65 | 314.22 |
| 1.43 | 1.45 | 3.72 | 4.13 | 111.58 |
| 6.07 | 1.44 | 3.04 | 4.19 | 327.23 |
| 2.41 | 3.35 | 6.85 | 3.64 | 140.23 |
| 1.03 | 4.30 | 5.87 | 3.98 | 97.50 |
| 2.80 | 2.61 | 4.25 | 3.88 | 115.62 |
| 2.71 | 2.30 | 7.80 | 4.02 | 155.12 |
| 6.49 | 3.15 | 2.72 | 3.68 | 360.99 |
| 3.37 | 3.92 | 2.79 | 3.97 | 138.06 |
| 8.14 | 2.86 | 4.94 | 4.19 | 674.04 |
| 2.71 | 2.97 | 4.59 | 3.76 | 120.52 |
| 5.97 | 2.21 | 3.96 | 4.03 | 319.85 |
| 7.21 | 3.20 | 3.49 | 3.95 | 489.68 |
| 1.82 | 2.68 | 7.52 | 4.08 | 137.45 |
| 9.98 | 1.62 | 4.55 | 3.72 | 1088.51 |
| 7.45 | 2.70 | 4.22 | 4.18 | 536.13 |
| 5.71 | 2.10 | 3.77 | 4.30 | 300.71 |
| 5.81 | 2.42 | 4.31 | 3.63 | 321.51 |
| 1.12 | 2.98 | 5.69 | 4.22 | 119.86 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.62 | 3.12 | 7.03 | 3.86 | 799.06 |
| 9.09 | 2.36 | 4.33 | 4.24 | 877.45 |
| 2.03 | 2.41 | 6.88 | 3.80 | 128.79 |
| 9.44 | 2.70 | 7.94 | 3.74 | 1009.36 |
| 5.99 | 1.96 | 6.13 | 3.66 | 326.85 |
| 5.35 | 2.05 | 6.37 | 4.23 | 283.76 |
| 5.90 | 1.93 | 6.22 | 3.91 | 343.19 |
| 7.32 | 3.10 | 6.50 | 3.67 | 532.57 |
| 7.83 | 3.46 | 6.63 | 3.77 | 644.06 |
| 7.48 | 4.07 | 4.41 | 3.97 | 526.45 |
| 9.73 | 3.09 | 7.87 | 3.78 | 1080.12 |
| 3.87 | 3.68 | 2.16 | 3.87 | 151.90 |
| 6.77 | 2.84 | 3.58 | 3.91 | 399.06 |
| 6.81 | 3.41 | 2.19 | 3.92 | 423.27 |
| 9.21 | 2.62 | 6.20 | 4.13 | 917.37 |
| 1.58 | 3.82 | 5.03 | 4.27 | 137.81 |
| 1.54 | 2.50 | 5.73 | 4.00 | 135.15 |
| 6.35 | 2.25 | 4.61 | 4.38 | 392.80 |
| 8.24 | 3.52 | 4.63 | 3.84 | 694.28 |
| 4.96 | 2.80 | 3.63 | 4.23 | 217.27 |
| 5.29 | 2.40 | 6.28 | 4.12 | 289.47 |
| 4.25 | 2.44 | 3.52 | 3.83 | 173.11 |
| 3.49 | 2.94 | 4.18 | 3.95 | 152.06 |
| 5.59 | 2.05 | 5.69 | 4.33 | 304.20 |
| 7.97 | 3.02 | 6.57 | 3.99 | 658.66 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 6.31 | 3.43 | 3.22 | 4.04 | 361.47 |
| 9.40 | 3.26 | 3.11 | 4.22 | 952.67 |
| 1.10 | 3.11 | 2.59 | 4.17 | 103.75 |
| 8.67 | 3.89 | 4.65 | 3.87 | 773.67 |
| 1.02 | 2.28 | 2.60 | 4.15 | 86.61 |
| 9.98 | 1.62 | 4.55 | 3.72 | 1088.51 |
| 6.76 | 3.08 | 7.54 | 4.40 | 475.07 |
| 1.94 | 2.46 | 3.15 | 3.97 | 87.02 |
| 4.26 | 2.58 | 6.54 | 4.05 | 205.11 |
| 7.14 | 2.48 | 3.67 | 3.92 | 478.04 |
| 6.34 | 2.69 | 6.51 | 4.05 | 400.19 |
| 2.27 | 3.46 | 2.07 | 4.31 | 102.08 |
| 5.24 | 2.95 | 6.34 | 3.65 | 272.98 |
| 4.18 | 3.26 | 6.52 | 3.66 | 218.27 |
| 6.88 | 3.69 | 6.42 | 4.11 | 499.42 |
| 7.41 | 2.27 | 5.85 | 3.97 | 538.15 |
| 1.88 | 3.33 | 2.09 | 3.82 | 85.69 |
| 6.69 | 2.46 | 3.20 | 4.13 | 424.55 |
| 9.31 | 2.44 | 4.86 | 3.70 | 916.60 |
| 7.85 | 2.10 | 5.86 | 3.67 | 606.73 |
| 5.53 | 2.73 | 7.84 | 3.65 | 314.22 |
| 9.19 | 2.09 | 3.92 | 3.71 | 896.59 |
| 3.20 | 2.21 | 2.79 | 3.67 | 127.84 |
| 8.02 | 2.54 | 5.01 | 3.85 | 640.83 |
| 5.65 | 1.92 | 7.79 | 3.87 | 329.69 |

**Model a**

| **Desired** | **Actual** | **SE** | **RMSE** |
|---|---|---|---|
| 361.47 | 306.93 | 2974.33 | 70.55 |
| 952.67 | 851.79 | 10175.98 | |
| 103.75 | 113.15 | 88.37 | |
| 773.67 | 767.24 | 41.38 | |
| 86.61 | 116.42 | 888.35 | |
| 1088.51 | 1020.71 | 4596.67 | |
| 475.07 | 412.76 | 3882.73 | |
| 87.02 | 121.18 | 1167.22 | |
| 205.11 | 181.52 | 556.35 | |
| 478.04 | 545.02 | 4486.16 | |
| 400.19 | 391.48 | 75.81 | |
| 102.08 | 117.66 | 242.65 | |
| 272.98 | 257.62 | 235.89 | |
| 218.27 | 171.07 | 2227.83 | |
| 499.42 | 415.79 | 6993.95 | |
| 538.15 | 659.31 | 14679.56 | |
| 85.69 | 116.59 | 954.61 | |
| 424.55 | 406.17 | 337.90 | |
| 916.60 | 964.05 | 2251.42 | |
| 606.73 | 817.42 | 44388.63 | |
| 314.22 | 326.02 | 139.24 | |
| 896.59 | 956.62 | 3604.16 | |
| 127.84 | 141.96 | 199.24 | |
| 640.83 | 775.47 | 18128.69 | |
| 329.69 | 363.29 | 1128.84 | |

**Model b**

| **Actual** | **Desired** | **SE** | **RMSE** |
|---|---|---|---|
| 361.47 | 320.87 | 1648.60 | 79.51 |
| 952.67 | 735.41 | 47202.44 | |
| 103.75 | 148.16 | 1972.63 | |
| 773.67 | 656.59 | 13707.29 | |
| 86.61 | 149.33 | 3934.17 | |
| 1088.51 | 990.65 | 9576.53 | |
| 475.07 | 537.26 | 3867.57 | |
| 87.02 | 153.39 | 4405.15 | |
| 205.11 | 206.88 | 3.12 | |
| 478.04 | 442.75 | 1245.40 | |
| 400.19 | 413.96 | 189.59 | |
| 102.08 | 155.28 | 2830.49 | |
| 272.98 | 255.65 | 300.25 | |
| 218.27 | 200.80 | 305.37 | |
| 499.42 | 508.24 | 77.76 | |
| 538.15 | 652.54 | 13084.50 | |
| 85.69 | 151.05 | 4271.50 | |
| 424.55 | 344.31 | 6439.06 | |
| 916.60 | 931.76 | 229.80 | |
| 606.73 | 776.54 | 28836.91 | |
| 314.22 | 332.26 | 325.41 | |
| 896.59 | 911.85 | 232.72 | |
| 127.84 | 162.28 | 1186.21 | |
| 640.83 | 720.82 | 6398.22 | |
| 329.69 | 405.80 | 5792.73 | |

**Average**
**RMSE:** 75.03

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.27 | 3.16 | 7.77 | 4.05 | 724.34 |
| 6.58 | 2.46 | 4.98 | 4.20 | 410.24 |
| 9.04 | 2.62 | 6.14 | 4.22 | 902.75 |
| 4.95 | 2.19 | 7.52 | 3.91 | 259.22 |
| 1.34 | 1.95 | 6.24 | 4.45 | 117.52 |
| 2.09 | 3.02 | 4.92 | 4.11 | 107.54 |
| 1.00 | 2.16 | 3.47 | 4.39 | 106.17 |
| 9.13 | 2.33 | 2.08 | 4.09 | 860.20 |
| 4.40 | 3.31 | 6.71 | 4.16 | 219.63 |
| 5.19 | 2.46 | 4.78 | 3.62 | 246.75 |
| 5.71 | 2.10 | 3.77 | 4.30 | 300.71 |
| 8.77 | 4.23 | 4.43 | 4.30 | 819.25 |
| 7.44 | 2.71 | 6.85 | 3.79 | 540.47 |
| 7.80 | 3.30 | 3.44 | 3.82 | 567.99 |
| 1.03 | 4.30 | 5.87 | 3.98 | 97.50 |
| 1.22 | 2.12 | 7.94 | 4.05 | 152.47 |
| 8.51 | 2.60 | 6.10 | 3.86 | 735.82 |
| 8.79 | 2.34 | 2.60 | 3.94 | 775.37 |
| 4.95 | 2.93 | 2.03 | 3.90 | 227.50 |
| 1.03 | 2.71 | 7.57 | 4.01 | 139.51 |
| 7.35 | 2.74 | 5.48 | 3.85 | 517.29 |
| 3.54 | 2.41 | 5.33 | 3.86 | 175.59 |
| 8.22 | 2.40 | 7.91 | 3.92 | 720.06 |
| 7.44 | 3.41 | 3.78 | 3.68 | 552.02 |
| 6.43 | 3.24 | 6.32 | 4.16 | 392.85 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.82 | 2.28 | 3.04 | 4.21 | 101.58 |
| 1.88 | 2.97 | 3.54 | 4.10 | 98.27 |
| 2.71 | 2.56 | 3.29 | 3.89 | 116.91 |
| 7.44 | 3.41 | 3.78 | 3.68 | 552.02 |
| 2.58 | 2.40 | 3.15 | 3.77 | 86.91 |
| 5.78 | 3.07 | 3.38 | 4.11 | 298.74 |
| 2.72 | 2.85 | 3.26 | 4.25 | 122.93 |
| 1.12 | 2.98 | 5.69 | 4.22 | 119.86 |
| 8.62 | 3.12 | 7.03 | 3.86 | 799.06 |
| 3.81 | 4.02 | 6.65 | 4.07 | 187.34 |
| 7.64 | 2.57 | 5.99 | 4.33 | 569.96 |
| 4.05 | 2.40 | 6.34 | 3.84 | 192.66 |
| 2.37 | 2.93 | 7.66 | 3.96 | 155.45 |
| 3.39 | 3.12 | 5.69 | 3.77 | 165.89 |
| 2.67 | 3.16 | 5.41 | 4.32 | 157.98 |
| 9.40 | 3.26 | 3.11 | 4.22 | 952.67 |
| 6.58 | 2.46 | 4.98 | 4.20 | 410.24 |
| 4.04 | 3.00 | 4.57 | 3.89 | 159.03 |
| 1.23 | 2.93 | 3.59 | 4.04 | 92.39 |
| 4.41 | 2.40 | 5.68 | 3.96 | 224.79 |
| 1.54 | 2.50 | 5.73 | 4.00 | 135.15 |
| 8.79 | 3.26 | 6.04 | 3.89 | 818.72 |
| 1.02 | 2.28 | 2.60 | 4.15 | 86.61 |
| 7.92 | 2.87 | 7.65 | 3.75 | 652.28 |
| 6.66 | 2.87 | 4.50 | 3.94 | 406.59 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.22 | 3.24 | 5.99 | 4.15 | 699.75 |
| 8.10 | 3.33 | 3.27 | 4.17 | 656.30 |
| 9.04 | 2.76 | 3.89 | 3.61 | 842.89 |
| 3.54 | 2.41 | 5.33 | 3.86 | 175.59 |
| 3.53 | 4.04 | 7.01 | 4.16 | 180.45 |
| 7.30 | 1.78 | 3.01 | 3.95 | 489.89 |
| 1.20 | 2.63 | 2.91 | 3.91 | 99.59 |
| 6.81 | 2.36 | 2.02 | 3.98 | 429.73 |
| 6.63 | 2.81 | 3.02 | 4.00 | 401.24 |
| 5.80 | 3.19 | 6.18 | 4.28 | 320.89 |
| 4.33 | 2.77 | 7.60 | 3.93 | 228.76 |
| 9.94 | 3.42 | 2.39 | 3.99 | 1090.88 |
| 1.89 | 2.73 | 5.97 | 4.05 | 124.24 |
| 9.97 | 2.67 | 3.03 | 3.68 | 1101.66 |
| 9.73 | 3.09 | 7.87 | 3.78 | 1080.12 |
| 8.27 | 3.16 | 7.77 | 4.05 | 724.34 |
| 6.35 | 2.25 | 4.61 | 4.38 | 392.80 |
| 5.90 | 2.74 | 5.91 | 3.89 | 322.75 |
| 7.02 | 2.87 | 5.05 | 4.12 | 468.41 |
| 6.86 | 2.48 | 7.98 | 3.90 | 485.61 |
| 8.12 | 3.05 | 7.96 | 4.07 | 714.80 |
| 1.43 | 1.45 | 3.72 | 4.13 | 111.58 |
| 7.64 | 2.57 | 5.99 | 4.33 | 569.96 |
| 4.26 | 2.58 | 6.54 | 4.05 | 205.11 |
| 4.54 | 3.42 | 4.22 | 4.14 | 207.60 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 699.75 | 755.98 | 3161.88 | 99.53 |
| 656.30 | 743.53 | 7608.41 | |
| 842.89 | 833.01 | 97.55 | |
| 175.59 | 182.35 | 45.69 | |
| 180.45 | 163.91 | 273.71 | |
| 489.89 | 665.47 | 30827.91 | |
| 99.59 | 140.46 | 1670.19 | |
| 429.73 | 516.87 | 7593.82 | |
| 401.24 | 449.70 | 2348.44 | |
| 320.89 | 310.73 | 103.23 | |
| 228.76 | 213.45 | 234.41 | |
| 1090.88 | 852.13 | 57003.37 | |
| 124.24 | 145.59 | 455.65 | |
| 1101.66 | 853.10 | 61782.07 | |
| 1080.12 | 848.51 | 53642.14 | |
| 724.34 | 759.17 | 1213.35 | |
| 392.80 | 425.16 | 1046.95 | |
| 322.75 | 342.68 | 405.02 | |
| 468.41 | 524.09 | 3100.64 | |
| 485.61 | 502.24 | 276.54 | |
| 714.80 | 739.83 | 626.33 | |
| 111.58 | 144.67 | 1095.01 | |
| 569.96 | 684.01 | 13008.22 | |
| 205.11 | 209.76 | 21.63 | |
| 207.60 | 205.23 | 5.62 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 699.75 | 794.26 | 8932.77 | 95.60 |
| 656.30 | 817.16 | 25875.95 | |
| 842.89 | 775.62 | 4525.35 | |
| 175.59 | 139.56 | 1298.44 | |
| 180.45 | 192.23 | 138.72 | |
| 489.89 | 360.04 | 16860.51 | |
| 99.59 | 128.92 | 860.10 | |
| 429.73 | 297.19 | 17567.82 | |
| 401.24 | 357.87 | 1881.32 | |
| 320.89 | 447.10 | 15928.16 | |
| 228.76 | 195.78 | 1087.54 | |
| 1090.88 | 926.27 | 27096.87 | |
| 124.24 | 133.05 | 77.64 | |
| 1101.66 | 872.32 | 52596.04 | |
| 1080.12 | 877.50 | 41055.18 | |
| 724.34 | 754.18 | 890.38 | |
| 392.80 | 451.35 | 3427.62 | |
| 322.75 | 287.73 | 1226.60 | |
| 468.41 | 543.05 | 5571.65 | |
| 485.61 | 460.51 | 630.01 | |
| 714.80 | 704.67 | 102.58 | |
| 111.58 | 132.84 | 451.96 | |
| 569.96 | 581.32 | 129.04 | |
| 205.11 | 193.88 | 126.08 | |
| 207.60 | 196.09 | 132.46 | |

Average
RMSE: 97.56

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 2.22 | 2.67 | 4.56 | 3.99 | 135.65 |
| 3.87 | 2.82 | 7.71 | 3.59 | 200.84 |
| 6.05 | 2.43 | 7.73 | 3.89 | 377.36 |
| 2.50 | 2.75 | 3.60 | 4.25 | 118.98 |
| 8.32 | 3.42 | 4.42 | 4.26 | 695.90 |
| 1.59 | 2.64 | 7.05 | 4.19 | 154.13 |
| 4.11 | 2.48 | 3.10 | 3.95 | 152.17 |
| 1.88 | 2.97 | 3.54 | 4.10 | 98.27 |
| 1.58 | 3.82 | 5.03 | 4.27 | 137.81 |
| 9.63 | 3.24 | 7.78 | 4.21 | 1073.54 |
| 8.57 | 2.74 | 4.38 | 3.87 | 750.31 |
| 5.80 | 3.19 | 6.18 | 4.28 | 320.89 |
| 2.40 | 2.89 | 3.99 | 3.74 | 106.42 |
| 8.08 | 3.38 | 2.91 | 3.94 | 636.97 |
| 9.38 | 2.87 | 2.23 | 4.15 | 932.80 |
| 3.81 | 4.02 | 6.65 | 4.07 | 187.34 |
| 7.35 | 2.76 | 4.69 | 3.95 | 510.36 |
| 8.49 | 2.91 | 6.83 | 3.98 | 747.72 |
| 4.04 | 3.00 | 4.57 | 3.89 | 159.03 |
| 9.16 | 2.58 | 6.61 | 4.13 | 905.39 |
| 2.30 | 2.83 | 5.23 | 3.86 | 129.63 |
| 7.85 | 3.27 | 5.56 | 4.08 | 604.44 |
| 3.37 | 2.36 | 6.35 | 3.88 | 159.26 |
| 4.52 | 3.28 | 4.83 | 4.19 | 218.05 |
| 7.07 | 2.70 | 5.37 | 4.05 | 488.96 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.50 | 3.21 | 5.39 | 4.03 | 120.95 |
| 4.03 | 2.67 | 4.25 | 4.59 | 198.81 |
| 2.51 | 3.25 | 3.76 | 4.01 | 140.73 |
| 9.09 | 3.84 | 2.34 | 3.85 | 865.06 |
| 1.00 | 2.16 | 3.47 | 4.39 | 106.17 |
| 8.21 | 3.37 | 3.97 | 4.37 | 681.74 |
| 4.74 | 2.77 | 7.84 | 3.81 | 237.71 |
| 8.62 | 2.39 | 3.45 | 4.51 | 760.18 |
| 8.08 | 3.38 | 2.91 | 3.94 | 636.97 |
| 2.43 | 2.48 | 7.42 | 3.77 | 147.10 |
| 3.80 | 3.05 | 2.47 | 3.97 | 166.03 |
| 9.98 | 2.42 | 2.72 | 4.24 | 1109.20 |
| 9.65 | 2.82 | 4.57 | 3.89 | 1006.50 |
| 6.81 | 2.36 | 2.02 | 3.98 | 429.73 |
| 1.03 | 2.95 | 6.41 | 3.96 | 122.22 |
| 2.30 | 2.83 | 5.23 | 3.86 | 129.63 |
| 7.32 | 3.18 | 3.53 | 3.78 | 514.00 |
| 4.58 | 2.70 | 4.58 | 4.02 | 209.27 |
| 6.77 | 3.16 | 5.13 | 4.05 | 447.51 |
| 8.71 | 2.65 | 3.60 | 3.76 | 755.05 |
| 6.67 | 2.06 | 3.08 | 3.82 | 414.27 |
| 8.27 | 3.16 | 7.77 | 4.05 | 724.34 |
| 4.11 | 2.48 | 3.10 | 3.95 | 152.17 |
| 7.13 | 3.26 | 5.23 | 4.11 | 494.21 |
| 7.80 | 3.20 | 7.46 | 3.92 | 610.80 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 3.01 | 2.61 | 2.93 | 4.15 | 126.62 |
| 6.76 | 2.85 | 4.47 | 3.85 | 427.85 |
| 8.59 | 2.94 | 4.56 | 3.94 | 771.73 |
| 9.53 | 3.01 | 3.22 | 3.98 | 988.66 |
| 4.56 | 3.58 | 3.13 | 3.58 | 196.64 |
| 2.27 | 3.46 | 2.07 | 4.31 | 102.08 |
| 2.54 | 3.47 | 2.65 | 3.76 | 115.95 |
| 2.41 | 3.35 | 6.85 | 3.64 | 140.23 |
| 9.13 | 2.33 | 2.08 | 4.09 | 860.20 |
| 4.64 | 2.85 | 7.54 | 3.89 | 219.22 |
| 6.16 | 2.56 | 6.60 | 3.95 | 388.12 |
| 4.06 | 3.89 | 5.30 | 3.90 | 182.68 |
| 5.93 | 4.10 | 4.77 | 3.75 | 326.03 |
| 2.40 | 2.06 | 2.59 | 4.43 | 126.01 |
| 9.69 | 2.94 | 3.13 | 3.98 | 1022.72 |
| 9.40 | 3.26 | 3.11 | 4.22 | 952.67 |
| 4.74 | 2.77 | 7.84 | 3.81 | 237.71 |
| 6.88 | 3.69 | 6.42 | 4.11 | 499.42 |
| 4.33 | 2.24 | 6.88 | 3.71 | 226.07 |
| 4.96 | 2.80 | 3.63 | 4.23 | 217.27 |
| 4.53 | 2.69 | 2.73 | 4.17 | 189.20 |
| 6.98 | 2.32 | 5.01 | 4.15 | 471.56 |
| 2.68 | 1.58 | 7.28 | 4.03 | 141.27 |
| 7.76 | 2.58 | 7.95 | 3.99 | 619.75 |
| 3.30 | 2.70 | 2.11 | 4.21 | 132.71 |

**Model a**

| **Desired** | **Actual** | **SE** | **RMSE** |
|---|---|---|---|
| 126.62 | 137.98 | 128.97 | 65.23 |
| 427.85 | 342.84 | 7226.53 | |
| 771.73 | 679.81 | 8448.87 | |
| 988.66 | 832.17 | 24488.75 | |
| 196.64 | 165.33 | 980.01 | |
| 102.08 | 144.06 | 1762.42 | |
| 115.95 | 134.89 | 358.66 | |
| 140.23 | 145.26 | 25.25 | |
| 860.20 | 758.96 | 10249.56 | |
| 219.22 | 224.16 | 24.44 | |
| 388.12 | 326.83 | 3756.55 | |
| 182.68 | 166.03 | 277.30 | |
| 326.03 | 282.16 | 1924.81 | |
| 126.01 | 136.85 | 117.58 | |
| 1022.72 | 853.74 | 28554.94 | |
| 952.67 | 856.45 | 9258.33 | |
| 237.71 | 241.55 | 14.76 | |
| 499.42 | 468.78 | 938.89 | |
| 226.07 | 207.39 | 348.95 | |
| 217.27 | 174.98 | 1788.68 | |
| 189.20 | 155.82 | 1114.54 | |
| 471.56 | 405.67 | 4340.91 | |
| 141.27 | 144.92 | 13.31 | |
| 619.75 | 606.89 | 165.37 | |
| 132.71 | 141.25 | 72.89 | |

**Model b**

| **Desired** | **Actual** | **SE** | **RMSE** |
|---|---|---|---|
| 126.62 | 133.74 | 50.70 | 55.51 |
| 427.85 | 317.62 | 12150.16 | |
| 771.73 | 775.90 | 17.40 | |
| 988.66 | 924.26 | 4147.51 | |
| 196.64 | 185.41 | 126.13 | |
| 102.08 | 130.24 | 793.00 | |
| 115.95 | 134.46 | 342.53 | |
| 140.23 | 139.04 | 1.41 | |
| 860.20 | 904.82 | 1990.78 | |
| 219.22 | 165.82 | 2852.05 | |
| 388.12 | 262.60 | 15755.11 | |
| 182.68 | 140.75 | 1757.78 | |
| 326.03 | 213.06 | 12761.31 | |
| 126.01 | 133.63 | 58.12 | |
| 1022.72 | 936.71 | 7397.35 | |
| 952.67 | 914.46 | 1460.08 | |
| 237.71 | 176.37 | 3762.45 | |
| 499.42 | 427.71 | 5142.06 | |
| 226.07 | 186.19 | 1590.62 | |
| 217.27 | 169.22 | 2308.77 | |
| 189.20 | 153.08 | 1304.75 | |
| 471.56 | 436.31 | 1242.21 | |
| 141.27 | 138.04 | 10.43 | |
| 619.75 | 618.04 | 2.94 | |
| 132.71 | 135.92 | 10.30 | |

**Average**
**RMSE:** 60.37

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.34 | 2.83 | 4.17 | 4.11 | 703.42 |
| 5.06 | 3.16 | 2.36 | 4.07 | 240.02 |
| 4.13 | 3.22 | 4.98 | 4.19 | 185.69 |
| 4.64 | 2.85 | 7.54 | 3.89 | 219.22 |
| 8.77 | 3.44 | 6.53 | 4.07 | 841.77 |
| 3.66 | 2.77 | 7.70 | 3.99 | 198.50 |
| 8.67 | 3.89 | 4.65 | 3.87 | 773.67 |
| 7.32 | 3.18 | 3.53 | 3.78 | 514.00 |
| 9.65 | 2.82 | 4.57 | 3.89 | 1006.50 |
| 7.76 | 2.58 | 7.95 | 3.99 | 619.75 |
| 5.57 | 2.89 | 6.41 | 4.19 | 311.78 |
| 4.53 | 2.69 | 2.73 | 4.17 | 189.20 |
| 1.12 | 2.98 | 5.69 | 4.22 | 119.86 |
| 2.26 | 2.93 | 6.61 | 3.83 | 146.87 |
| 8.02 | 2.54 | 5.01 | 3.85 | 640.83 |
| 3.53 | 4.04 | 7.01 | 4.16 | 180.45 |
| 3.20 | 2.21 | 2.79 | 3.67 | 127.84 |
| 4.74 | 2.77 | 7.84 | 3.81 | 237.71 |
| 6.24 | 2.50 | 7.89 | 4.16 | 413.48 |
| 3.81 | 4.02 | 6.65 | 4.07 | 187.34 |
| 3.14 | 2.63 | 3.95 | 4.36 | 139.79 |
| 5.13 | 2.44 | 3.46 | 3.84 | 246.42 |
| 2.51 | 3.25 | 3.76 | 4.01 | 140.73 |
| 7.41 | 2.77 | 3.64 | 4.31 | 563.17 |
| 5.19 | 2.46 | 4.78 | 3.62 | 246.75 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.12 | 3.05 | 7.96 | 4.07 | 714.80 |
| 9.13 | 2.33 | 2.08 | 4.09 | 860.20 |
| 6.76 | 2.85 | 4.47 | 3.85 | 427.85 |
| 6.21 | 3.16 | 5.22 | 3.69 | 365.43 |
| 6.92 | 2.99 | 5.15 | 4.04 | 466.84 |
| 1.70 | 2.86 | 4.19 | 4.13 | 91.37 |
| 8.24 | 3.52 | 4.63 | 3.84 | 694.28 |
| 2.23 | 2.65 | 4.42 | 3.97 | 109.83 |
| 8.12 | 3.12 | 2.24 | 4.10 | 646.90 |
| 7.76 | 2.58 | 7.95 | 3.99 | 619.75 |
| 5.93 | 2.62 | 5.23 | 3.75 | 343.92 |
| 5.81 | 2.42 | 4.31 | 3.63 | 321.51 |
| 9.98 | 1.62 | 4.55 | 3.72 | 1088.51 |
| 4.09 | 2.69 | 2.36 | 4.37 | 160.19 |
| 6.18 | 3.80 | 3.96 | 3.94 | 335.30 |
| 5.96 | 2.43 | 3.68 | 4.31 | 336.86 |
| 6.53 | 2.94 | 7.44 | 3.79 | 419.52 |
| 1.14 | 2.41 | 2.30 | 4.07 | 91.91 |
| 5.65 | 1.92 | 7.79 | 3.87 | 329.69 |
| 7.23 | 2.69 | 4.25 | 3.98 | 515.15 |
| 1.64 | 3.36 | 3.18 | 3.98 | 103.04 |
| 1.88 | 3.33 | 2.09 | 3.82 | 85.69 |
| 4.18 | 3.26 | 6.52 | 3.66 | 218.27 |
| 5.76 | 3.10 | 7.63 | 3.67 | 349.75 |
| 8.98 | 3.20 | 2.23 | 4.15 | 845.34 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 7.13 | 3.26 | 5.23 | 4.11 | 494.21 |
| 4.06 | 3.84 | 7.44 | 3.84 | 208.11 |
| 6.77 | 3.16 | 5.13 | 4.05 | 447.51 |
| 6.96 | 2.32 | 5.01 | 4.15 | 471.56 |
| 1.58 | 3.82 | 5.03 | 4.27 | 137.81 |
| 1.20 | 2.63 | 2.91 | 3.91 | 99.59 |
| 2.26 | 2.93 | 6.61 | 3.83 | 146.87 |
| 7.64 | 2.57 | 5.99 | 4.33 | 569.96 |
| 4.45 | 3.66 | 2.74 | 4.03 | 198.89 |
| 4.13 | 3.62 | 3.08 | 4.16 | 180.22 |
| 4.63 | 2.45 | 3.71 | 4.20 | 225.29 |
| 4.69 | 2.73 | 2.26 | 4.00 | 199.51 |
| 1.09 | 3.19 | 5.70 | 3.60 | 104.45 |
| 2.70 | 3.47 | 6.47 | 4.19 | 171.87 |
| 6.12 | 3.15 | 5.02 | 4.02 | 371.94 |
| 7.23 | 3.77 | 5.40 | 3.92 | 495.80 |
| 6.58 | 2.46 | 4.98 | 4.20 | 410.24 |
| 4.74 | 2.77 | 7.84 | 3.81 | 237.71 |
| 5.64 | 3.85 | 4.06 | 3.89 | 295.97 |
| 5.16 | 2.36 | 3.34 | 4.18 | 242.60 |
| 2.33 | 3.87 | 6.80 | 3.77 | 144.55 |
| 5.90 | 2.74 | 5.91 | 3.89 | 322.75 |
| 4.40 | 3.31 | 6.71 | 4.16 | 219.63 |
| 6.72 | 2.59 | 5.91 | 3.96 | 431.70 |
| 2.58 | 3.73 | 4.04 | 3.80 | 132.49 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 126.62 | 163.73 | 1377.09 | 44.07 |
| 427.85 | 416.82 | 121.66 | |
| 771.73 | 784.92 | 174.01 | |
| 988.66 | 885.40 | 10663.20 | |
| 196.64 | 200.71 | 16.60 | |
| 102.08 | 157.54 | 3075.65 | |
| 115.95 | 157.97 | 1765.66 | |
| 140.23 | 153.82 | 184.56 | |
| 860.20 | 813.30 | 2199.22 | |
| 219.22 | 245.26 | 678.08 | |
| 388.12 | 387.07 | 1.10 | |
| 182.68 | 187.02 | 18.83 | |
| 326.03 | 316.70 | 87.01 | |
| 126.01 | 161.65 | 1270.42 | |
| 1022.72 | 895.19 | 16263.49 | |
| 952.67 | 887.88 | 4198.23 | |
| 237.71 | 251.49 | 189.96 | |
| 499.42 | 544.11 | 1997.01 | |
| 226.07 | 203.69 | 500.76 | |
| 217.27 | 230.00 | 162.00 | |
| 189.20 | 200.64 | 130.84 | |
| 471.56 | 492.70 | 446.96 | |
| 141.27 | 179.61 | 1470.29 | |
| 619.75 | 634.44 | 215.88 | |
| 132.71 | 169.29 | 1338.45 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 126.62 | 141.40 | 218.33 | 33.29 |
| 427.85 | 436.56 | 75.93 | |
| 771.73 | 789.81 | 326.72 | |
| 988.66 | 919.61 | 4767.88 | |
| 196.64 | 212.94 | 265.62 | |
| 102.08 | 128.51 | 698.67 | |
| 115.95 | 137.79 | 477.20 | |
| 140.23 | 148.58 | 69.72 | |
| 860.20 | 898.73 | 1484.65 | |
| 219.22 | 224.34 | 26.19 | |
| 388.12 | 350.47 | 1417.48 | |
| 182.68 | 178.57 | 16.87 | |
| 326.03 | 325.60 | 0.19 | |
| 126.01 | 132.43 | 41.27 | |
| 1022.72 | 934.65 | 7756.04 | |
| 952.67 | 899.92 | 2782.72 | |
| 237.71 | 237.63 | 0.01 | |
| 499.42 | 417.35 | 6736.22 | |
| 226.07 | 215.65 | 108.64 | |
| 217.27 | 206.48 | 116.36 | |
| 189.20 | 184.74 | 19.92 | |
| 471.56 | 469.81 | 3.05 | |
| 141.27 | 153.76 | 156.10 | |
| 619.75 | 619.91 | 0.03 | |
| 132.71 | 144.43 | 137.42 | |

Average
RMSE 38.68

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 4.20 | 2.94 | 2.72 | 4.22 | 168.33 |
| 6.63 | 2.81 | 3.02 | 4.00 | 401.24 |
| 7.99 | 2.65 | 3.92 | 3.93 | 617.11 |
| 2.33 | 3.87 | 6.80 | 3.77 | 144.55 |
| 3.06 | 1.72 | 5.41 | 3.84 | 121.32 |
| 4.13 | 3.62 | 3.08 | 4.16 | 180.22 |
| 8.77 | 3.44 | 6.53 | 4.07 | 841.77 |
| 3.93 | 2.58 | 3.12 | 3.79 | 173.29 |
| 7.77 | 3.14 | 2.04 | 4.05 | 574.13 |
| 1.54 | 2.50 | 5.73 | 4.00 | 135.15 |
| 8.39 | 2.22 | 7.84 | 4.14 | 742.67 |
| 2.62 | 2.80 | 4.72 | 4.13 | 113.92 |
| 4.66 | 2.12 | 4.57 | 3.76 | 222.58 |
| 3.01 | 3.02 | 7.88 | 4.01 | 181.21 |
| 4.26 | 2.58 | 6.54 | 4.05 | 205.11 |
| 4.93 | 3.19 | 7.27 | 4.07 | 275.68 |
| 8.92 | 3.78 | 4.50 | 4.00 | 845.92 |
| 6.17 | 3.73 | 4.48 | 4.28 | 365.36 |
| 4.51 | 3.24 | 4.58 | 4.37 | 214.65 |
| 4.16 | 2.39 | 2.85 | 4.13 | 169.88 |
| 4.25 | 2.44 | 3.52 | 3.83 | 173.11 |
| 2.09 | 3.12 | 3.66 | 4.19 | 121.92 |
| 8.12 | 3.05 | 7.96 | 4.07 | 714.80 |
| 2.63 | 2.52 | 2.66 | 4.08 | 111.85 |
| 9.13 | 2.33 | 2.08 | 4.09 | 860.20 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 4.41 | 2.40 | 5.68 | 3.96 | 224.79 |
| 3.27 | 2.48 | 6.41 | 4.10 | 154.92 |
| 4.04 | 3.00 | 4.57 | 3.89 | 159.03 |
| 6.25 | 2.47 | 7.18 | 4.00 | 402.72 |
| 8.68 | 2.75 | 6.95 | 3.77 | 794.84 |
| 2.30 | 2.83 | 5.23 | 3.86 | 129.63 |
| 3.49 | 2.94 | 4.18 | 3.95 | 152.06 |
| 2.04 | 3.43 | 6.67 | 4.12 | 126.93 |
| 3.07 | 3.32 | 7.01 | 3.99 | 167.51 |
| 1.69 | 2.50 | 7.91 | 3.62 | 144.89 |
| 6.18 | 3.80 | 3.96 | 3.94 | 335.30 |
| 1.80 | 3.60 | 7.93 | 4.06 | 161.77 |
| 5.90 | 2.74 | 5.91 | 3.89 | 322.75 |
| 7.85 | 2.10 | 5.86 | 3.67 | 606.73 |
| 5.95 | 2.61 | 3.14 | 3.89 | 304.78 |
| 1.77 | 2.45 | 2.18 | 3.91 | 76.79 |
| 6.01 | 1.91 | 2.78 | 3.64 | 315.36 |
| 6.81 | 2.22 | 7.11 | 4.00 | 469.58 |
| 4.81 | 3.20 | 2.97 | 4.26 | 212.89 |
| 2.09 | 3.12 | 3.66 | 4.19 | 121.92 |
| 4.40 | 3.31 | 6.71 | 4.16 | 219.63 |
| 8.86 | 2.95 | 4.85 | 4.04 | 821.76 |
| 8.22 | 3.24 | 5.99 | 4.15 | 699.75 |
| 4.87 | 2.86 | 2.32 | 3.96 | 202.25 |
| 2.03 | 2.41 | 6.88 | 3.80 | 128.79 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 3.72 | 2.77 | 2.05 | 3.92 | 133.14 |
| 8.34 | 2.83 | 4.17 | 4.11 | 703.42 |
| 3.98 | 2.46 | 4.45 | 3.72 | 151.11 |
| 8.96 | 2.93 | 6.40 | 3.99 | 862.69 |
| 9.19 | 2.09 | 3.92 | 3.71 | 896.59 |
| 8.57 | 2.74 | 4.38 | 3.87 | 750.31 |
| 2.36 | 2.62 | 2.93 | 4.16 | 130.54 |
| 4.85 | 2.70 | 2.50 | 4.09 | 217.86 |
| 6.39 | 1.83 | 5.54 | 3.76 | 379.46 |
| 4.66 | 2.12 | 4.57 | 3.76 | 222.58 |
| 5.13 | 2.44 | 3.46 | 3.84 | 246.42 |
| 1.26 | 2.08 | 6.08 | 3.81 | 127.72 |
| 8.10 | 3.33 | 3.27 | 4.17 | 656.30 |
| 6.69 | 2.10 | 4.66 | 3.76 | 429.47 |
| 2.58 | 2.40 | 3.15 | 3.77 | 86.91 |
| 1.82 | 2.28 | 3.04 | 4.21 | 101.58 |
| 6.16 | 2.56 | 6.60 | 3.95 | 388.12 |
| 6.18 | 3.80 | 3.96 | 3.94 | 335.30 |
| 7.92 | 2.24 | 3.47 | 3.65 | 602.92 |
| 7.92 | 2.87 | 7.65 | 3.75 | 652.28 |
| 1.63 | 1.81 | 7.10 | 3.99 | 150.85 |
| 4.08 | 2.88 | 2.43 | 4.13 | 162.23 |
| 4.32 | 1.82 | 4.19 | 3.68 | 170.50 |
| 4.63 | 2.88 | 4.35 | 4.21 | 206.65 |
| 9.22 | 3.69 | 3.22 | 3.77 | 904.94 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 133.14 | 148.85 | 246.81 | 64.23 |
| 703.42 | 727.54 | 581.72 | |
| 151.11 | 167.97 | 284.33 | |
| 862.69 | 808.20 | 2968.80 | |
| 896.59 | 734.16 | 26381.94 | |
| 750.31 | 710.26 | 1604.19 | |
| 130.54 | 134.70 | 17.29 | |
| 217.86 | 199.11 | 351.71 | |
| 379.46 | 336.03 | 1886.26 | |
| 222.58 | 206.85 | 247.30 | |
| 246.42 | 231.61 | 219.21 | |
| 127.72 | 132.49 | 22.75 | |
| 656.30 | 737.14 | 6535.36 | |
| 429.47 | 349.28 | 6430.80 | |
| 86.91 | 136.24 | 2433.50 | |
| 101.58 | 132.75 | 971.56 | |
| 388.12 | 344.97 | 1861.53 | |
| 335.30 | 346.26 | 120.06 | |
| 602.92 | 429.54 | 30060.40 | |
| 652.28 | 540.25 | 12549.69 | |
| 150.85 | 133.86 | 288.54 | |
| 162.23 | 159.57 | 7.10 | |
| 170.50 | 181.98 | 131.85 | |
| 206.65 | 201.37 | 27.93 | |
| 904.94 | 821.81 | 6911.13 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 133.14 | 155.44 | 497.15 | 53.13 |
| 703.42 | 728.68 | 638.16 | |
| 151.11 | 158.68 | 57.34 | |
| 862.69 | 777.64 | 7233.09 | |
| 896.59 | 747.41 | 22254.67 | |
| 750.31 | 752.38 | 4.30 | |
| 130.54 | 148.72 | 330.66 | |
| 217.86 | 191.23 | 709.06 | |
| 379.46 | 402.65 | 537.58 | |
| 222.58 | 189.49 | 1094.66 | |
| 246.42 | 207.73 | 1497.26 | |
| 127.72 | 146.38 | 348.07 | |
| 656.30 | 744.13 | 7713.66 | |
| 429.47 | 435.56 | 37.07 | |
| 86.91 | 148.48 | 3790.74 | |
| 101.58 | 147.11 | 2072.91 | |
| 388.12 | 404.11 | 255.62 | |
| 335.30 | 350.14 | 220.30 | |
| 602.92 | 636.12 | 1102.43 | |
| 652.28 | 733.35 | 6572.92 | |
| 150.85 | 147.14 | 13.75 | |
| 162.23 | 162.83 | 0.36 | |
| 170.50 | 176.34 | 34.15 | |
| 206.65 | 191.16 | 240.02 | |
| 904.94 | 789.61 | 13302.14 | |

Average
RMSE: 58.68

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.62 | 2.39 | 3.45 | 4.51 | 760.18 |
| 4.89 | 1.97 | 6.99 | 3.91 | 262.07 |
| 8.34 | 2.83 | 4.17 | 4.11 | 703.42 |
| 8.96 | 2.93 | 6.40 | 3.99 | 862.69 |
| 8.19 | 3.21 | 2.35 | 3.93 | 640.93 |
| 4.69 | 3.32 | 5.68 | 3.99 | 227.61 |
| 1.61 | 4.14 | 4.67 | 3.84 | 98.04 |
| 4.64 | 2.85 | 7.54 | 3.89 | 219.22 |
| 2.58 | 3.73 | 4.04 | 3.80 | 132.49 |
| 1.25 | 3.11 | 3.94 | 4.22 | 96.51 |
| 2.81 | 2.05 | 7.65 | 3.96 | 174.24 |
| 9.44 | 2.70 | 7.94 | 3.74 | 1009.36 |
| 4.05 | 2.40 | 6.34 | 3.84 | 192.66 |
| 4.81 | 3.20 | 2.97 | 4.26 | 212.89 |
| 1.92 | 3.59 | 3.46 | 4.04 | 102.97 |
| 2.58 | 2.40 | 3.15 | 3.77 | 86.91 |
| 3.37 | 2.36 | 6.35 | 3.88 | 159.26 |
| 8.08 | 3.38 | 2.91 | 3.94 | 636.97 |
| 2.30 | 2.83 | 5.23 | 3.86 | 129.63 |
| 3.53 | 4.04 | 7.01 | 4.16 | 180.45 |
| 8.45 | 2.72 | 3.40 | 3.71 | 713.60 |
| 9.25 | 2.52 | 7.57 | 3.90 | 962.33 |
| 1.25 | 3.32 | 2.40 | 4.15 | 111.27 |
| 8.10 | 3.33 | 3.27 | 4.17 | 656.30 |
| 1.59 | 2.64 | 7.05 | 4.19 | 154.13 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.99 | 2.72 | 2.87 | 3.76 | 101.76 |
| 6.53 | 2.94 | 7.44 | 3.79 | 419.52 |
| 1.20 | 2.63 | 2.91 | 3.91 | 99.59 |
| 3.81 | 2.39 | 6.20 | 3.68 | 185.03 |
| 9.46 | 3.59 | 7.02 | 3.80 | 1003.16 |
| 8.12 | 3.12 | 2.24 | 4.10 | 646.90 |
| 9.22 | 3.69 | 3.22 | 3.77 | 904.94 |
| 7.83 | 3.46 | 6.63 | 3.77 | 644.06 |
| 6.07 | 1.44 | 3.04 | 4.19 | 327.23 |
| 9.19 | 2.09 | 3.92 | 3.71 | 896.59 |
| 8.91 | 3.88 | 5.30 | 4.14 | 867.33 |
| 2.68 | 1.58 | 7.28 | 4.03 | 141.27 |
| 1.63 | 1.81 | 7.10 | 3.99 | 150.85 |
| 1.09 | 3.19 | 5.70 | 3.60 | 104.45 |
| 8.90 | 2.79 | 2.94 | 3.96 | 812.38 |
| 2.03 | 2.41 | 6.88 | 3.80 | 128.79 |
| 5.94 | 2.92 | 5.02 | 4.02 | 316.19 |
| 8.77 | 2.32 | 6.92 | 3.76 | 812.19 |
| 4.45 | 3.66 | 2.74 | 4.03 | 198.89 |
| 2.81 | 2.05 | 7.65 | 3.96 | 174.24 |
| 8.73 | 2.09 | 5.30 | 4.19 | 772.98 |
| 5.91 | 2.78 | 4.61 | 3.90 | 347.85 |
| 8.08 | 3.38 | 2.91 | 3.94 | 636.97 |
| 6.22 | 3.11 | 7.27 | 4.14 | 391.53 |
| 3.80 | 3.05 | 2.47 | 3.97 | 166.03 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 3.35 | 2.92 | 7.55 | 4.14 | 173.45 |
| 7.07 | 2.70 | 5.37 | 4.05 | 488.96 |
| 3.87 | 2.45 | 4.23 | 4.11 | 172.26 |
| 6.77 | 2.84 | 3.58 | 3.91 | 399.06 |
| 9.46 | 3.59 | 7.02 | 3.80 | 1003.16 |
| 6.00 | 3.25 | 5.47 | 3.85 | 350.77 |
| 5.46 | 2.83 | 4.06 | 3.82 | 277.11 |
| 2.68 | 1.58 | 7.28 | 4.03 | 141.27 |
| 7.38 | 2.92 | 3.04 | 3.93 | 514.12 |
| 5.13 | 2.44 | 3.46 | 3.84 | 246.42 |
| 1.82 | 2.68 | 7.52 | 4.08 | 137.45 |
| 6.18 | 3.80 | 3.96 | 3.94 | 335.30 |
| 8.76 | 3.30 | 7.47 | 4.31 | 842.59 |
| 3.54 | 2.41 | 5.33 | 3.86 | 175.59 |
| 1.12 | 2.98 | 5.69 | 4.22 | 119.86 |
| 1.54 | 2.50 | 5.73 | 4.00 | 135.15 |
| 8.19 | 3.21 | 2.35 | 3.93 | 640.93 |
| 7.99 | 2.65 | 3.92 | 3.93 | 617.11 |
| 5.85 | 2.13 | 5.26 | 3.92 | 312.96 |
| 5.85 | 3.31 | 3.00 | 4.02 | 311.22 |
| 3.87 | 2.82 | 7.71 | 3.59 | 200.84 |
| 8.18 | 2.85 | 6.03 | 4.25 | 694.60 |
| 1.00 | 2.16 | 3.47 | 4.39 | 106.17 |
| 7.67 | 2.45 | 5.98 | 3.73 | 579.69 |
| 9.38 | 2.87 | 2.23 | 4.15 | 932.80 |

Model a

| *Desired* | *Actual* | *SE* | *RMSE* |
|---|---|---|---|
| 173.45 | 132.69 | 1661.15 | 115.75 |
| 488.96 | 354.41 | 18103.31 | |
| 172.26 | 147.30 | 622.97 | |
| 399.06 | 301.31 | 9554.54 | |
| 1003.16 | 758.91 | 59657.91 | |
| 350.77 | 180.22 | 29086.18 | |
| 277.11 | 159.56 | 13817.24 | |
| 141.27 | 189.38 | 2314.14 | |
| 514.12 | 432.65 | 6636.72 | |
| 246.42 | 167.89 | 6167.56 | |
| 137.45 | 127.15 | 106.13 | |
| 335.30 | 227.40 | 11643.33 | |
| 842.59 | 588.66 | 64479.36 | |
| 175.59 | 137.87 | 1422.49 | |
| 119.86 | 125.58 | 32.70 | |
| 135.15 | 127.38 | 60.36 | |
| 640.93 | 623.11 | 317.46 | |
| 617.11 | 574.22 | 1839.74 | |
| 312.96 | 242.26 | 4999.20 | |
| 311.22 | 200.02 | 12366.46 | |
| 200.84 | 132.23 | 4708.01 | |
| 694.60 | 495.44 | 39664.82 | |
| 106.17 | 133.86 | 766.90 | |
| 579.69 | 469.88 | 12057.33 | |
| 932.80 | 751.54 | 32856.45 | |

Model b

| *Desired* | *Actual* | *SE* | *RMSE* |
|---|---|---|---|
| 173.45 | 226.63 | 2828.07 | 36.41 |
| 488.96 | 471.00 | 322.61 | |
| 172.26 | 194.14 | 478.91 | |
| 399.06 | 421.30 | 494.78 | |
| 1003.16 | 935.65 | 4558.25 | |
| 350.77 | 388.48 | 1422.27 | |
| 277.11 | 311.51 | 1183.02 | |
| 141.27 | 156.67 | 237.15 | |
| 514.12 | 510.64 | 12.09 | |
| 246.42 | 265.89 | 379.20 | |
| 137.45 | 139.17 | 2.96 | |
| 335.30 | 418.01 | 6840.88 | |
| 842.59 | 883.14 | 1644.51 | |
| 175.59 | 169.38 | 38.61 | |
| 119.86 | 132.37 | 156.43 | |
| 135.15 | 132.22 | 8.57 | |
| 640.93 | 709.73 | 4733.55 | |
| 617.11 | 650.75 | 1131.78 | |
| 312.96 | 338.25 | 639.66 | |
| 311.22 | 353.80 | 1813.18 | |
| 200.84 | 231.37 | 932.11 | |
| 694.60 | 740.84 | 2138.06 | |
| 106.17 | 131.74 | 653.59 | |
| 579.69 | 585.46 | 33.24 | |
| 932.80 | 911.47 | 454.81 | |

| | |
|---|---|
| **Average** | |
| *RMSE* | 76.08 |

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 3.27 | 2.48 | 6.41 | 4.10 | 154.92 |
| 7.38 | 2.92 | 3.04 | 3.93 | 514.12 |
| 9.65 | 2.82 | 4.57 | 3.89 | 1006.50 |
| 5.93 | 4.10 | 4.77 | 3.75 | 326.03 |
| 7.63 | 2.07 | 4.49 | 4.21 | 560.03 |
| 8.03 | 2.39 | 5.80 | 3.98 | 655.40 |
| 3.42 | 2.56 | 5.49 | 3.95 | 168.57 |
| 3.47 | 2.21 | 7.44 | 4.13 | 175.75 |
| 8.49 | 2.91 | 6.83 | 3.98 | 747.72 |
| 5.93 | 2.62 | 5.23 | 3.75 | 343.92 |
| 5.96 | 2.43 | 3.68 | 4.31 | 336.86 |
| 5.91 | 2.78 | 4.61 | 3.90 | 347.85 |
| 7.83 | 3.46 | 6.63 | 3.77 | 644.06 |
| 7.60 | 3.21 | 5.00 | 4.10 | 585.09 |
| 3.37 | 3.92 | 2.79 | 3.97 | 138.06 |
| 7.20 | 3.42 | 5.54 | 4.20 | 526.04 |
| 6.81 | 3.41 | 2.19 | 3.92 | 423.27 |
| 6.63 | 2.81 | 3.02 | 4.00 | 401.24 |
| 4.16 | 2.39 | 2.85 | 4.13 | 169.88 |
| 4.44 | 2.81 | 7.31 | 4.34 | 238.50 |
| 7.23 | 2.69 | 4.25 | 3.98 | 515.15 |
| 9.16 | 2.58 | 6.61 | 4.13 | 905.39 |
| 5.64 | 3.85 | 4.06 | 3.89 | 295.97 |
| 6.86 | 2.48 | 7.98 | 3.90 | 485.61 |
| 9.44 | 2.70 | 7.94 | 3.74 | 1009.36 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 5.94 | 2.92 | 5.02 | 4.02 | 316.19 |
| 4.52 | 3.28 | 4.83 | 4.19 | 218.05 |
| 3.68 | 2.51 | 6.68 | 4.01 | 182.91 |
| 1.64 | 2.59 | 5.65 | 3.96 | 137.57 |
| 9.58 | 2.61 | 2.89 | 3.50 | 977.06 |
| 3.07 | 3.32 | 7.01 | 3.99 | 167.51 |
| 4.74 | 2.77 | 7.84 | 3.81 | 237.71 |
| 2.23 | 2.65 | 4.42 | 3.97 | 109.83 |
| 6.69 | 2.18 | 4.44 | 3.95 | 394.49 |
| 3.14 | 2.63 | 3.95 | 4.36 | 139.79 |
| 9.65 | 2.82 | 4.57 | 3.89 | 1006.50 |
| 9.24 | 2.36 | 2.25 | 3.93 | 899.95 |
| 9.42 | 3.31 | 5.79 | 4.04 | 967.23 |
| 2.50 | 2.75 | 3.60 | 4.25 | 118.98 |
| 9.94 | 3.42 | 2.39 | 3.99 | 1090.88 |
| 8.91 | 3.88 | 5.30 | 4.14 | 867.33 |
| 5.53 | 2.73 | 7.84 | 3.65 | 314.22 |
| 2.59 | 2.54 | 2.57 | 4.08 | 107.74 |
| 8.18 | 2.85 | 6.03 | 4.25 | 694.60 |
| 1.89 | 2.73 | 5.97 | 4.05 | 124.24 |
| 2.71 | 2.97 | 4.59 | 3.76 | 120.52 |
| 9.98 | 1.62 | 4.55 | 3.72 | 1088.51 |
| 7.20 | 3.42 | 5.54 | 4.20 | 526.04 |
| 7.07 | 2.58 | 6.43 | 3.90 | 484.24 |
| 2.58 | 3.73 | 4.04 | 3.80 | 132.49 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 9.46 | 3.59 | 7.02 | 3.80 | 1003.16 |
| 3.64 | 3.14 | 4.96 | 4.02 | 164.56 |
| 6.86 | 2.48 | 7.98 | 3.90 | 485.61 |
| 2.39 | 2.21 | 5.46 | 3.60 | 121.95 |
| 5.57 | 2.89 | 6.41 | 4.19 | 311.78 |
| 2.26 | 3.22 | 2.91 | 4.06 | 97.67 |
| 9.94 | 3.42 | 2.39 | 3.99 | 1090.88 |
| 1.92 | 3.59 | 3.46 | 4.04 | 102.97 |
| 1.80 | 3.60 | 7.93 | 4.06 | 161.77 |
| 1.77 | 2.45 | 2.18 | 3.91 | 76.79 |
| 8.27 | 3.16 | 7.77 | 4.05 | 724.34 |
| 2.38 | 2.65 | 5.00 | 3.63 | 127.63 |
| 3.43 | 3.49 | 2.68 | 3.78 | 147.47 |
| 9.34 | 2.41 | 3.80 | 4.06 | 925.54 |
| 4.06 | 3.89 | 5.30 | 3.90 | 182.68 |
| 9.09 | 2.36 | 4.33 | 4.24 | 877.45 |
| 4.85 | 2.70 | 2.50 | 4.09 | 217.86 |
| 2.01 | 2.30 | 3.95 | 4.09 | 106.91 |
| 6.63 | 2.81 | 3.02 | 4.00 | 401.24 |
| 1.71 | 3.26 | 3.05 | 3.79 | 83.17 |
| 8.96 | 2.93 | 6.40 | 3.99 | 862.69 |
| 7.67 | 2.45 | 5.98 | 3.73 | 579.69 |
| 6.92 | 2.99 | 5.15 | 4.04 | 466.84 |
| 8.09 | 3.55 | 5.27 | 3.79 | 656.89 |
| 1.76 | 2.84 | 4.68 | 4.11 | 116.72 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 1003.16 | 960.55 | 1815.28 | 65.89 |
| 164.56 | 200.70 | 1305.87 | |
| 485.61 | 506.55 | 438.41 | |
| 121.95 | 232.19 | 12153.32 | |
| 311.78 | 321.85 | 101.37 | |
| 97.67 | 183.62 | 7387.82 | |
| 1090.88 | 974.14 | 13627.63 | |
| 102.97 | 180.63 | 6031.58 | |
| 161.77 | 181.19 | 377.22 | |
| 76.79 | 187.55 | 12267.22 | |
| 724.34 | 793.24 | 4747.06 | |
| 127.63 | 208.88 | 6601.00 | |
| 147.47 | 198.28 | 2582.11 | |
| 925.54 | 914.66 | 118.30 | |
| 182.68 | 206.92 | 587.48 | |
| 877.45 | 877.67 | 0.05 | |
| 217.86 | 227.92 | 101.30 | |
| 106.91 | 187.20 | 6445.90 | |
| 401.24 | 427.83 | 707.16 | |
| 83.17 | 184.49 | 10265.75 | |
| 862.69 | 895.05 | 1047.35 | |
| 579.69 | 614.14 | 1186.86 | |
| 466.84 | 504.94 | 1451.37 | |
| 656.89 | 770.26 | 12851.98 | |
| 116.72 | 182.67 | 4350.05 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 1003.16 | 997.94 | 27.22 | 88.08 |
| 164.56 | 134.32 | 914.36 | |
| 485.61 | 353.64 | 17417.10 | |
| 121.95 | 155.24 | 1108.22 | |
| 311.78 | 214.69 | 9425.80 | |
| 97.67 | 119.02 | 455.63 | |
| 1090.88 | 1020.95 | 4889.76 | |
| 102.97 | 117.67 | 216.00 | |
| 161.77 | 118.28 | 1891.46 | |
| 76.79 | 121.10 | 1963.31 | |
| 724.34 | 784.76 | 3650.37 | |
| 127.63 | 139.29 | 136.00 | |
| 147.47 | 153.93 | 41.76 | |
| 925.54 | 981.47 | 3127.93 | |
| 182.68 | 198.00 | 234.82 | |
| 877.45 | 904.99 | 758.65 | |
| 217.86 | 194.65 | 538.80 | |
| 106.91 | 125.35 | 339.91 | |
| 401.24 | 572.01 | 29163.23 | |
| 83.17 | 117.18 | 1156.70 | |
| 862.69 | 945.53 | 6861.98 | |
| 579.69 | 747.60 | 28193.96 | |
| 466.84 | 546.72 | 6381.02 | |
| 656.89 | 930.83 | 75045.83 | |
| 116.72 | 119.15 | 5.92 | |

Average
RMSE: 76.99

# APPENDIX E: MLR TRAINING AND TESTING DATA AND ESTIMATED Y-VALUES FOR FUNCTION 1

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.77 | 2.57 | 2.94 | 4.14 | 783.10 |
| 1.61 | 4.14 | 4.67 | 3.84 | 98.04 |
| 7.45 | 2.70 | 4.22 | 4.18 | 536.13 |
| 7.25 | 2.12 | 4.41 | 4.11 | 485.37 |
| 3.01 | 3.02 | 7.88 | 4.01 | 181.21 |
| 2.81 | 2.05 | 7.65 | 3.96 | 174.24 |
| 1.10 | 2.66 | 2.30 | 3.80 | 97.42 |
| 8.59 | 2.94 | 4.56 | 3.94 | 771.73 |
| 3.79 | 2.29 | 2.32 | 4.11 | 141.22 |
| 6.69 | 2.10 | 4.66 | 3.76 | 429.47 |
| 5.34 | 2.61 | 2.83 | 4.09 | 256.90 |
| 5.64 | 2.04 | 6.58 | 4.11 | 338.63 |
| 5.53 | 2.73 | 7.84 | 3.65 | 314.22 |
| 1.77 | 2.45 | 2.18 | 3.91 | 76.79 |
| 7.32 | 3.18 | 3.53 | 3.78 | 514.00 |
| 8.41 | 3.44 | 7.77 | 3.82 | 758.43 |
| 2.38 | 3.10 | 7.54 | 4.04 | 168.96 |
| 2.77 | 2.52 | 2.47 | 4.00 | 113.58 |
| 7.19 | 3.42 | 2.06 | 3.85 | 469.72 |
| 4.06 | 3.89 | 5.30 | 3.90 | 182.68 |
| 2.20 | 3.63 | 2.06 | 4.20 | 105.80 |
| 4.18 | 2.69 | 3.25 | 3.83 | 150.17 |
| 5.78 | 3.07 | 3.38 | 4.11 | 298.74 |
| 4.23 | 3.47 | 7.52 | 3.89 | 234.37 |
| 3.30 | 2.70 | 2.11 | 4.21 | 132.71 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.62 | 3.12 | 7.03 | 3.86 | 799.06 |
| 2.54 | 3.47 | 2.65 | 3.76 | 115.95 |
| 9.38 | 2.87 | 2.23 | 4.15 | 932.80 |
| 1.88 | 3.33 | 2.09 | 3.82 | 85.69 |
| 3.87 | 2.82 | 7.71 | 3.59 | 200.84 |
| 7.92 | 2.87 | 7.65 | 3.75 | 652.28 |
| 2.09 | 3.12 | 3.66 | 4.19 | 121.92 |
| 2.72 | 2.31 | 7.64 | 3.64 | 154.07 |
| 8.91 | 3.88 | 5.30 | 4.14 | 867.33 |
| 6.22 | 3.81 | 6.85 | 4.25 | 404.60 |
| 7.45 | 2.70 | 4.22 | 4.18 | 536.13 |
| 6.53 | 1.92 | 4.06 | 3.67 | 391.17 |
| 2.26 | 2.93 | 6.61 | 3.83 | 146.87 |
| 4.54 | 3.42 | 4.22 | 4.14 | 207.60 |
| 2.63 | 2.52 | 2.66 | 4.08 | 111.85 |
| 8.51 | 2.60 | 6.10 | 3.86 | 735.82 |
| 7.21 | 3.20 | 3.49 | 3.95 | 489.68 |
| 7.67 | 2.45 | 5.98 | 3.73 | 579.69 |
| 1.10 | 3.11 | 2.59 | 4.17 | 103.75 |
| 6.89 | 2.63 | 6.15 | 3.94 | 459.86 |
| 3.71 | 2.65 | 3.12 | 4.31 | 149.62 |
| 8.17 | 3.27 | 4.85 | 4.06 | 702.34 |
| 1.99 | 2.72 | 2.87 | 3.76 | 101.76 |
| 8.79 | 3.26 | 6.04 | 3.89 | 818.72 |
| 1.05 | 3.42 | 7.46 | 4.23 | 138.90 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 7.77 | 3.14 | 2.04 | 4.05 | 574.13 |
| 9.53 | 2.20 | 4.82 | 4.22 | 978.95 |
| 8.23 | 2.71 | 5.68 | 4.10 | 688.11 |
| 3.27 | 2.48 | 6.41 | 4.10 | 154.92 |
| 5.42 | 2.91 | 2.12 | 4.17 | 263.19 |
| 1.25 | 3.11 | 3.94 | 4.22 | 96.51 |
| 7.63 | 2.07 | 4.49 | 4.21 | 560.03 |
| 1.54 | 2.50 | 5.73 | 4.00 | 135.15 |
| 2.42 | 2.83 | 2.67 | 3.69 | 82.99 |
| 7.23 | 2.69 | 4.25 | 3.98 | 515.15 |
| 8.45 | 2.72 | 3.40 | 3.71 | 713.60 |
| 2.59 | 2.54 | 2.57 | 4.08 | 107.74 |
| 4.69 | 2.73 | 2.26 | 4.00 | 199.51 |
| 9.37 | 2.33 | 5.91 | 4.09 | 960.29 |
| 9.75 | 2.09 | 4.58 | 4.14 | 1059.86 |
| 5.65 | 2.10 | 5.47 | 3.74 | 311.22 |
| 8.71 | 2.65 | 3.60 | 3.76 | 755.05 |
| 5.90 | 1.93 | 6.22 | 3.91 | 343.19 |
| 4.41 | 2.40 | 5.68 | 3.96 | 224.79 |
| 1.94 | 2.46 | 3.15 | 3.97 | 87.02 |
| 3.14 | 1.77 | 7.10 | 4.20 | 176.23 |
| 6.22 | 3.11 | 7.27 | 4.14 | 391.53 |
| 1.38 | 2.94 | 2.64 | 4.24 | 107.69 |
| 4.32 | 1.82 | 4.19 | 3.68 | 170.50 |
| 3.81 | 2.39 | 6.20 | 3.68 | 185.03 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 574.13 | 570.85 | 10.80 | 114.18 |
| 978.95 | 726.57 | 63695.51 | |
| 688.11 | 611.43 | 5880.26 | |
| 154.92 | 173.19 | 333.89 | |
| 263.19 | 363.25 | 10010.66 | |
| 96.51 | -5.50 | 10407.35 | |
| 560.03 | 558.75 | 1.65 | |
| 135.15 | 20.13 | 13229.35 | |
| 82.99 | 98.16 | 230.21 | |
| 515.15 | 523.18 | 64.51 | |
| 713.60 | 630.53 | 6900.12 | |
| 107.74 | 112.84 | 26.00 | |
| 199.51 | 298.77 | 9853.16 | |
| 960.29 | 712.59 | 61355.73 | |
| 1059.86 | 746.11 | 98440.11 | |
| 311.22 | 383.11 | 5169.40 | |
| 755.05 | 653.49 | 10312.76 | |
| 343.19 | 405.44 | 3875.50 | |
| 224.79 | 273.94 | 2415.87 | |
| 87.02 | 55.64 | 984.60 | |
| 176.23 | 161.41 | 219.63 | |
| 391.53 | 433.75 | 1782.63 | |
| 107.69 | 6.28 | 10285.33 | |
| 170.50 | 265.98 | 9117.87 | |
| 185.03 | 221.11 | 1301.59 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 574.13 | 636.78 | 3923.82 | 103.29 |
| 978.95 | 805.82 | 29975.54 | |
| 688.11 | 680.82 | 53.09 | |
| 154.92 | 205.10 | 2518.73 | |
| 263.19 | 411.42 | 21970.11 | |
| 96.51 | 11.13 | 7290.68 | |
| 560.03 | 623.64 | 4045.99 | |
| 135.15 | 38.96 | 9253.55 | |
| 82.99 | 123.66 | 1654.07 | |
| 515.15 | 585.03 | 4883.35 | |
| 713.60 | 701.56 | 144.88 | |
| 107.74 | 139.59 | 1014.56 | |
| 199.51 | 341.43 | 20140.95 | |
| 960.29 | 790.64 | 28781.76 | |
| 1059.86 | 827.02 | 54212.45 | |
| 311.22 | 432.98 | 14827.38 | |
| 755.05 | 726.49 | 815.51 | |
| 343.19 | 457.22 | 13003.37 | |
| 224.79 | 314.47 | 8043.05 | |
| 87.02 | 77.50 | 90.59 | |
| 176.23 | 192.31 | 258.78 | |
| 391.53 | 487.95 | 9297.03 | |
| 107.69 | 23.91 | 7018.79 | |
| 170.50 | 305.84 | 18316.57 | |
| 185.03 | 257.12 | 5197.00 | |

Average
RMSE: 108.73

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.56 | 3.42 | 6.67 | 4.17 | 107.76 |
| 6.68 | 2.84 | 7.80 | 4.07 | 470.40 |
| 2.53 | 2.84 | 3.62 | 3.86 | 118.78 |
| 7.92 | 2.87 | 7.65 | 3.75 | 652.28 |
| 6.34 | 2.69 | 6.51 | 4.05 | 400.19 |
| 4.44 | 2.81 | 7.31 | 4.34 | 238.50 |
| 6.94 | 2.99 | 5.65 | 4.13 | 458.89 |
| 6.48 | 3.75 | 2.15 | 4.09 | 370.25 |
| 9.24 | 2.36 | 2.25 | 3.93 | 899.95 |
| 4.08 | 2.88 | 2.43 | 4.13 | 162.23 |
| 7.75 | 2.32 | 7.12 | 4.21 | 604.77 |
| 1.50 | 3.21 | 5.39 | 4.03 | 120.95 |
| 1.87 | 1.86 | 3.57 | 3.96 | 101.99 |
| 3.44 | 2.82 | 3.44 | 4.14 | 151.33 |
| 4.14 | 3.24 | 6.01 | 4.00 | 204.42 |
| 9.37 | 2.33 | 5.91 | 4.09 | 960.29 |
| 7.32 | 3.10 | 6.50 | 3.67 | 532.57 |
| 4.51 | 3.24 | 4.58 | 4.37 | 214.65 |
| 8.79 | 2.34 | 2.60 | 3.94 | 775.37 |
| 6.63 | 2.81 | 3.02 | 4.00 | 401.24 |
| 2.38 | 2.65 | 5.00 | 3.63 | 127.63 |
| 1.58 | 3.82 | 5.03 | 4.27 | 137.81 |
| 7.85 | 2.10 | 5.86 | 3.67 | 606.73 |
| 4.13 | 3.62 | 3.08 | 4.16 | 180.22 |
| 7.30 | 3.33 | 3.82 | 3.57 | 493.15 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.98 | 3.20 | 2.23 | 4.15 | 845.34 |
| 5.89 | 2.80 | 7.61 | 4.02 | 356.75 |
| 7.25 | 2.12 | 4.41 | 4.11 | 485.37 |
| 8.14 | 2.86 | 4.94 | 4.19 | 674.04 |
| 1.38 | 2.94 | 2.64 | 4.24 | 107.69 |
| 1.26 | 3.04 | 4.15 | 3.77 | 101.87 |
| 6.19 | 3.02 | 4.69 | 3.99 | 357.59 |
| 6.72 | 2.59 | 5.91 | 3.96 | 431.70 |
| 8.99 | 2.87 | 3.86 | 4.43 | 859.51 |
| 1.36 | 2.85 | 7.31 | 3.92 | 141.27 |
| 6.49 | 3.15 | 2.72 | 3.68 | 360.99 |
| 5.29 | 2.40 | 6.28 | 4.12 | 289.47 |
| 6.00 | 3.25 | 5.47 | 3.85 | 350.77 |
| 2.80 | 2.61 | 4.25 | 3.88 | 115.62 |
| 8.40 | 2.54 | 5.21 | 3.52 | 736.81 |
| 6.16 | 2.56 | 6.60 | 3.95 | 388.12 |
| 5.34 | 2.61 | 2.83 | 4.09 | 256.90 |
| 7.65 | 2.96 | 4.69 | 4.07 | 550.36 |
| 7.48 | 4.07 | 4.41 | 3.97 | 526.45 |
| 5.32 | 2.94 | 4.26 | 4.11 | 252.90 |
| 5.59 | 2.05 | 5.69 | 4.33 | 304.20 |
| 2.50 | 2.75 | 3.60 | 4.25 | 118.98 |
| 7.80 | 3.20 | 7.46 | 3.92 | 610.80 |
| 9.82 | 2.83 | 7.30 | 3.94 | 1098.06 |
| 8.12 | 3.12 | 2.24 | 4.10 | 646.90 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 7.44 | 2.71 | 6.85 | 3.79 | 540.47 |
| 8.18 | 2.85 | 6.03 | 4.25 | 694.60 |
| 6.94 | 2.99 | 5.65 | 4.13 | 458.89 |
| 4.44 | 2.81 | 7.31 | 4.34 | 238.50 |
| 5.62 | 2.13 | 2.21 | 3.95 | 269.32 |
| 1.76 | 2.84 | 4.68 | 4.11 | 116.72 |
| 4.33 | 2.77 | 7.60 | 3.93 | 228.76 |
| 9.54 | 2.54 | 2.23 | 4.02 | 983.71 |
| 5.64 | 3.85 | 4.06 | 3.89 | 295.97 |
| 3.64 | 3.14 | 4.96 | 4.02 | 164.56 |
| 9.81 | 3.18 | 2.62 | 3.70 | 1050.38 |
| 6.22 | 3.81 | 6.85 | 4.25 | 404.60 |
| 9.21 | 2.62 | 6.20 | 4.13 | 917.37 |
| 6.95 | 3.17 | 7.54 | 3.98 | 489.65 |
| 6.31 | 3.43 | 3.22 | 4.04 | 361.47 |
| 6.18 | 3.80 | 3.96 | 3.94 | 335.30 |
| 4.54 | 3.42 | 4.22 | 4.14 | 207.60 |
| 1.38 | 2.35 | 2.05 | 3.99 | 90.93 |
| 5.85 | 2.13 | 5.26 | 3.92 | 312.96 |
| 9.46 | 3.59 | 7.02 | 3.80 | 1003.16 |
| 4.95 | 2.93 | 2.03 | 3.90 | 227.50 |
| 3.49 | 2.94 | 4.18 | 3.95 | 152.06 |
| 8.56 | 4.07 | 3.63 | 4.24 | 772.36 |
| 8.84 | 2.01 | 4.07 | 4.00 | 811.16 |
| 4.16 | 3.51 | 4.58 | 4.26 | 179.11 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 540.47 | 565.04 | 603.68 | 52.98 |
| 694.60 | 672.49 | 488.63 | |
| 458.89 | 498.64 | 1580.23 | |
| 238.50 | 232.64 | 34.39 | |
| 269.32 | 343.17 | 5452.75 | |
| 116.72 | 77.79 | 1515.96 | |
| 228.76 | 223.57 | 26.90 | |
| 983.71 | 897.48 | 7434.30 | |
| 295.97 | 345.56 | 2459.59 | |
| 164.56 | 172.47 | 62.54 | |
| 1050.38 | 946.17 | 10858.21 | |
| 404.60 | 410.11 | 30.38 | |
| 917.37 | 840.14 | 5963.42 | |
| 489.65 | 499.45 | 95.98 | |
| 361.47 | 420.88 | 3529.84 | |
| 335.30 | 404.88 | 4840.94 | |
| 207.60 | 241.17 | 1127.25 | |
| 90.93 | 66.86 | 579.08 | |
| 312.96 | 368.38 | 3072.12 | |
| 1003.16 | 884.47 | 14086.57 | |
| 227.50 | 277.69 | 2519.10 | |
| 152.06 | 162.75 | 114.30 | |
| 772.36 | 732.20 | 1612.58 | |
| 811.16 | 778.12 | 1091.94 | |
| 179.11 | 210.65 | 995.29 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 540.47 | 565.29 | 616.15 | 46.97 |
| 694.60 | 677.49 | 292.61 | |
| 458.89 | 495.96 | 1374.31 | |
| 238.50 | 218.21 | 411.86 | |
| 269.32 | 333.62 | 4133.79 | |
| 116.72 | 56.52 | 3624.77 | |
| 228.76 | 208.74 | 400.69 | |
| 983.71 | 912.42 | 5081.53 | |
| 295.97 | 336.12 | 1612.17 | |
| 164.56 | 155.38 | 84.28 | |
| 1050.38 | 963.26 | 7588.93 | |
| 404.60 | 403.52 | 1.16 | |
| 917.37 | 852.55 | 4201.44 | |
| 489.65 | 496.80 | 51.16 | |
| 361.47 | 414.76 | 2840.57 | |
| 335.30 | 398.06 | 3938.15 | |
| 207.60 | 227.12 | 381.07 | |
| 90.93 | 45.11 | 2099.24 | |
| 312.96 | 359.95 | 2208.28 | |
| 1003.16 | 898.83 | 10883.51 | |
| 227.50 | 265.25 | 1425.07 | |
| 152.06 | 145.24 | 46.60 | |
| 772.36 | 739.84 | 1057.57 | |
| 811.16 | 787.78 | 546.61 | |
| 179.11 | 195.25 | 260.71 | |

| Average RMSE: | 49.98 |
|---|---|

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 4.95 | 2.19 | 7.52 | 3.91 | 259.22 |
| 3.68 | 2.51 | 6.68 | 4.01 | 182.91 |
| 7.45 | 2.91 | 7.35 | 3.84 | 558.55 |
| 4.16 | 3.51 | 4.58 | 4.26 | 179.11 |
| 5.84 | 2.76 | 6.85 | 3.93 | 341.32 |
| 9.58 | 2.61 | 2.89 | 3.50 | 977.06 |
| 2.70 | 3.47 | 6.47 | 4.19 | 171.87 |
| 6.53 | 2.94 | 7.44 | 3.79 | 419.52 |
| 8.65 | 2.76 | 5.24 | 4.19 | 793.63 |
| 3.80 | 3.05 | 2.47 | 3.97 | 166.03 |
| 5.85 | 3.31 | 3.00 | 4.02 | 311.22 |
| 2.09 | 3.12 | 3.66 | 4.19 | 121.92 |
| 1.87 | 1.86 | 3.57 | 3.96 | 101.99 |
| 9.40 | 3.26 | 3.11 | 4.22 | 952.67 |
| 4.04 | 3.00 | 4.57 | 3.89 | 159.03 |
| 5.50 | 2.75 | 6.77 | 4.18 | 308.08 |
| 8.02 | 2.32 | 4.10 | 4.09 | 618.56 |
| 6.72 | 2.59 | 5.91 | 3.96 | 431.70 |
| 3.20 | 2.21 | 2.79 | 3.67 | 127.84 |
| 5.00 | 2.42 | 5.69 | 3.80 | 245.03 |
| 3.54 | 2.41 | 5.33 | 3.86 | 175.59 |
| 4.54 | 3.42 | 4.22 | 4.14 | 207.60 |
| 3.47 | 2.21 | 7.44 | 4.13 | 175.75 |
| 3.14 | 2.63 | 3.95 | 4.36 | 139.79 |
| 4.13 | 3.62 | 3.08 | 4.16 | 180.22 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 5.65 | 1.92 | 7.79 | 3.87 | 329.69 |
| 6.81 | 2.36 | 2.02 | 3.98 | 429.73 |
| 9.38 | 2.87 | 2.23 | 4.15 | 932.80 |
| 5.89 | 2.80 | 7.61 | 4.02 | 356.75 |
| 2.38 | 2.80 | 6.35 | 3.77 | 138.34 |
| 3.68 | 2.51 | 6.68 | 4.01 | 182.91 |
| 6.12 | 3.15 | 5.02 | 4.02 | 371.94 |
| 5.69 | 2.05 | 2.27 | 3.94 | 271.61 |
| 9.21 | 2.62 | 6.20 | 4.13 | 917.37 |
| 8.10 | 3.33 | 3.27 | 4.17 | 656.30 |
| 7.53 | 3.02 | 6.41 | 4.18 | 569.89 |
| 6.63 | 2.81 | 3.02 | 4.00 | 401.24 |
| 7.14 | 2.48 | 3.67 | 3.92 | 478.04 |
| 7.41 | 2.77 | 3.64 | 4.31 | 563.17 |
| 8.51 | 2.60 | 6.10 | 3.86 | 735.82 |
| 1.03 | 2.95 | 6.41 | 3.96 | 122.22 |
| 8.40 | 2.54 | 5.21 | 3.52 | 736.81 |
| 3.72 | 2.77 | 2.05 | 3.92 | 133.14 |
| 9.44 | 2.70 | 7.94 | 3.74 | 1009.36 |
| 7.32 | 3.10 | 6.50 | 3.67 | 532.57 |
| 4.58 | 2.70 | 4.58 | 4.02 | 209.27 |
| 6.33 | 2.00 | 4.49 | 3.71 | 360.77 |
| 7.36 | 2.98 | 5.70 | 3.85 | 548.95 |
| 7.45 | 2.70 | 4.22 | 4.18 | 536.13 |
| 7.44 | 3.41 | 3.78 | 3.68 | 552.02 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.88 | 3.33 | 2.09 | 3.82 | 85.69 |
| 7.07 | 2.58 | 6.43 | 3.90 | 484.24 |
| 6.43 | 3.24 | 6.32 | 4.16 | 392.85 |
| 8.99 | 2.87 | 3.86 | 4.43 | 859.51 |
| 3.72 | 2.77 | 2.05 | 3.92 | 133.14 |
| 2.40 | 2.06 | 2.59 | 4.43 | 126.01 |
| 9.53 | 2.20 | 4.82 | 4.22 | 978.95 |
| 1.88 | 2.97 | 3.54 | 4.10 | 98.27 |
| 1.09 | 3.19 | 5.70 | 3.60 | 104.45 |
| 6.24 | 2.72 | 3.59 | 3.87 | 337.23 |
| 8.22 | 2.40 | 7.91 | 3.92 | 720.06 |
| 6.58 | 2.46 | 4.98 | 4.20 | 410.24 |
| 9.44 | 2.70 | 7.94 | 3.74 | 1009.36 |
| 2.67 | 3.16 | 5.41 | 4.32 | 157.98 |
| 1.71 | 3.26 | 3.05 | 3.79 | 83.17 |
| 7.13 | 3.26 | 5.23 | 4.11 | 494.21 |
| 4.17 | 1.75 | 4.63 | 3.78 | 196.33 |
| 1.31 | 3.38 | 6.57 | 4.02 | 148.29 |
| 5.86 | 2.86 | 4.45 | 3.80 | 311.17 |
| 8.39 | 2.22 | 7.84 | 4.14 | 742.67 |
| 4.66 | 2.12 | 4.57 | 3.76 | 222.58 |
| 3.37 | 3.92 | 2.79 | 3.97 | 138.06 |
| 8.73 | 2.09 | 5.30 | 4.19 | 772.98 |
| 5.50 | 2.37 | 3.80 | 4.04 | 289.91 |
| 2.80 | 2.61 | 4.25 | 3.88 | 115.62 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 85.69 | 99.58 | 193.09 | 12.12 |
| 484.24 | 484.77 | 0.28 | |
| 392.85 | 401.02 | 66.74 | |
| 859.51 | 852.72 | 46.07 | |
| 133.14 | 147.07 | 194.06 | |
| 126.01 | 123.52 | 6.20 | |
| 978.95 | 995.67 | 279.34 | |
| 98.27 | 113.53 | 233.09 | |
| 104.45 | 113.69 | 85.37 | |
| 337.23 | 347.76 | 110.82 | |
| 720.06 | 708.96 | 123.31 | |
| 410.24 | 408.64 | 2.56 | |
| 1009.36 | 995.19 | 200.63 | |
| 157.98 | 146.06 | 142.12 | |
| 83.17 | 101.42 | 332.89 | |
| 494.21 | 487.03 | 51.46 | |
| 196.33 | 180.24 | 259.04 | |
| 148.29 | 134.06 | 202.51 | |
| 311.17 | 309.96 | 1.46 | |
| 742.67 | 748.96 | 39.59 | |
| 222.58 | 208.29 | 204.02 | |
| 138.06 | 137.95 | 0.01 | |
| 772.98 | 795.64 | 513.68 | |
| 289.91 | 275.37 | 211.61 | |
| 115.62 | 128.65 | 170.01 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 85.69 | 107.05 | 456.43 | 15.01 |
| 484.24 | 492.11 | 61.84 | |
| 392.85 | 400.69 | 61.50 | |
| 859.51 | 856.11 | 11.56 | |
| 133.14 | 152.86 | 388.56 | |
| 126.01 | 116.33 | 93.79 | |
| 978.95 | 1005.49 | 704.11 | |
| 98.27 | 113.44 | 230.18 | |
| 104.45 | 123.47 | 361.60 | |
| 337.23 | 356.96 | 389.31 | |
| 720.06 | 716.82 | 10.50 | |
| 410.24 | 409.41 | 0.69 | |
| 1009.36 | 1011.74 | 5.68 | |
| 157.98 | 139.24 | 351.28 | |
| 83.17 | 109.19 | 676.64 | |
| 494.21 | 490.92 | 10.80 | |
| 196.33 | 187.96 | 70.15 | |
| 148.29 | 132.74 | 241.77 | |
| 311.17 | 319.36 | 67.10 | |
| 742.67 | 752.06 | 88.30 | |
| 222.58 | 217.00 | 31.09 | |
| 138.06 | 142.13 | 16.59 | |
| 772.98 | 802.35 | 862.74 | |
| 289.91 | 279.19 | 114.99 | |
| 115.62 | 133.56 | 322.10 | |

Average
RMSE: 13.56

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 2.96 | 3.38 | 5.88 | 4.21 | 147.88 |
| 9.19 | 2.09 | 3.92 | 3.71 | 896.59 |
| 4.44 | 2.81 | 7.31 | 4.34 | 238.50 |
| 9.13 | 2.33 | 2.08 | 4.09 | 860.20 |
| 1.54 | 2.50 | 5.73 | 4.00 | 135.15 |
| 6.24 | 2.50 | 7.89 | 4.16 | 413.48 |
| 1.20 | 2.63 | 2.91 | 3.91 | 99.59 |
| 9.26 | 2.25 | 4.46 | 4.32 | 918.17 |
| 2.71 | 2.30 | 7.80 | 4.02 | 155.12 |
| 5.35 | 2.05 | 6.37 | 4.23 | 283.76 |
| 8.18 | 2.85 | 6.03 | 4.25 | 694.60 |
| 1.35 | 2.50 | 4.74 | 3.91 | 102.81 |
| 4.58 | 2.70 | 4.58 | 4.02 | 209.27 |
| 8.56 | 4.07 | 3.63 | 4.24 | 772.36 |
| 8.02 | 2.32 | 4.10 | 4.09 | 618.56 |
| 9.31 | 2.44 | 4.86 | 3.70 | 916.60 |
| 9.24 | 2.36 | 2.25 | 3.93 | 899.95 |
| 8.98 | 3.20 | 2.23 | 4.15 | 845.34 |
| 6.23 | 3.15 | 6.72 | 3.41 | 369.31 |
| 7.41 | 2.77 | 3.64 | 4.31 | 563.17 |
| 4.96 | 2.80 | 3.63 | 4.23 | 217.27 |
| 3.42 | 2.56 | 5.49 | 3.95 | 168.57 |
| 2.67 | 3.16 | 5.41 | 4.32 | 157.98 |
| 1.08 | 2.38 | 7.05 | 3.95 | 130.26 |
| 7.17 | 2.93 | 5.61 | 4.04 | 505.13 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 3.14 | 2.63 | 3.95 | 4.36 | 139.79 |
| 1.61 | 4.14 | 4.67 | 3.84 | 98.04 |
| 3.27 | 2.48 | 6.41 | 4.10 | 154.92 |
| 4.69 | 2.73 | 2.26 | 4.00 | 199.51 |
| 8.23 | 2.71 | 5.68 | 4.10 | 688.11 |
| 1.10 | 2.66 | 2.30 | 3.80 | 97.42 |
| 9.08 | 2.48 | 6.53 | 4.00 | 864.17 |
| 9.21 | 2.62 | 6.20 | 4.13 | 917.37 |
| 2.37 | 2.93 | 7.66 | 3.96 | 155.45 |
| 8.77 | 3.44 | 6.53 | 4.07 | 841.77 |
| 3.38 | 3.44 | 3.07 | 3.63 | 123.79 |
| 6.92 | 2.31 | 7.01 | 4.22 | 483.61 |
| 2.68 | 1.58 | 7.28 | 4.03 | 141.27 |
| 2.77 | 2.52 | 2.47 | 4.00 | 113.58 |
| 3.64 | 3.14 | 4.96 | 4.02 | 164.56 |
| 4.61 | 3.25 | 4.73 | 3.86 | 222.29 |
| 5.62 | 2.13 | 2.21 | 3.95 | 269.32 |
| 2.56 | 3.17 | 7.54 | 4.15 | 155.10 |
| 2.33 | 3.87 | 6.80 | 3.77 | 144.55 |
| 4.64 | 2.85 | 7.54 | 3.89 | 219.22 |
| 9.22 | 3.69 | 3.22 | 3.77 | 904.94 |
| 7.17 | 3.63 | 3.83 | 4.19 | 507.21 |
| 6.94 | 2.99 | 5.65 | 4.13 | 458.89 |
| 5.90 | 1.93 | 6.22 | 3.91 | 343.19 |
| 7.75 | 2.32 | 7.12 | 4.21 | 604.77 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 4.04 | 3.00 | 4.57 | 3.89 | 159.03 |
| 2.27 | 3.46 | 2.07 | 4.31 | 102.08 |
| 7.35 | 2.74 | 5.48 | 3.85 | 517.29 |
| 1.64 | 3.36 | 3.18 | 3.98 | 103.04 |
| 7.13 | 3.26 | 5.23 | 4.11 | 494.21 |
| 6.44 | 2.58 | 7.46 | 3.71 | 413.33 |
| 8.65 | 2.76 | 5.24 | 4.19 | 793.63 |
| 2.72 | 2.31 | 7.64 | 3.64 | 154.07 |
| 2.70 | 3.47 | 6.47 | 4.19 | 171.87 |
| 1.94 | 2.46 | 3.15 | 3.97 | 87.02 |
| 9.44 | 2.70 | 7.94 | 3.74 | 1009.36 |
| 8.66 | 2.03 | 3.39 | 4.01 | 750.85 |
| 3.07 | 3.32 | 7.01 | 3.99 | 167.51 |
| 9.73 | 3.09 | 7.87 | 3.78 | 1080.12 |
| 9.82 | 2.83 | 7.30 | 3.94 | 1098.06 |
| 6.81 | 2.30 | 5.69 | 4.35 | 443.50 |
| 2.42 | 2.83 | 2.67 | 3.69 | 82.99 |
| 4.25 | 2.44 | 3.52 | 3.83 | 173.11 |
| 2.51 | 3.25 | 3.76 | 4.01 | 140.73 |
| 7.46 | 3.17 | 3.10 | 3.98 | 527.84 |
| 9.34 | 2.41 | 3.80 | 4.06 | 925.54 |
| 5.64 | 3.24 | 5.34 | 3.87 | 310.66 |
| 8.01 | 3.04 | 7.51 | 4.17 | 686.90 |
| 3.35 | 2.92 | 7.55 | 4.14 | 173.45 |
| 3.71 | 2.65 | 3.12 | 4.31 | 149.62 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 159.03 | 290.59 | 17308.53 | 143.87 |
| 102.08 | 158.18 | 3147.48 | |
| 517.29 | 546.05 | 827.12 | |
| 103.04 | 112.56 | 90.60 | |
| 494.21 | 528.75 | 1192.90 | |
| 413.33 | 474.86 | 3786.55 | |
| 793.63 | 648.00 | 21210.58 | |
| 154.07 | 191.23 | 1380.92 | |
| 171.87 | 189.64 | 315.86 | |
| 87.02 | 134.01 | 2208.65 | |
| 1009.36 | 711.03 | 89000.71 | |
| 750.85 | 648.82 | 10410.41 | |
| 167.51 | 217.36 | 2485.01 | |
| 1080.12 | 733.62 | 120060.78 | |
| 1098.06 | 740.80 | 127633.35 | |
| 443.50 | 503.84 | 3641.03 | |
| 82.99 | 169.28 | 7446.45 | |
| 173.11 | 306.59 | 17817.00 | |
| 140.73 | 176.18 | 1256.72 | |
| 527.84 | 554.33 | 701.55 | |
| 925.54 | 703.00 | 49524.71 | |
| 310.66 | 412.75 | 10421.22 | |
| 686.90 | 597.74 | 7948.43 | |
| 173.45 | 238.20 | 4192.62 | |
| 149.62 | 265.64 | 13460.86 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 159.03 | 244.04 | 7225.85 | 148.18 |
| 102.08 | 121.23 | 366.90 | |
| 517.29 | 504.17 | 172.12 | |
| 103.04 | 81.96 | 444.09 | |
| 494.21 | 485.84 | 70.07 | |
| 413.33 | 429.33 | 256.19 | |
| 793.63 | 613.89 | 32307.52 | |
| 154.07 | 150.81 | 10.67 | |
| 171.87 | 149.36 | 506.53 | |
| 87.02 | 100.18 | 173.34 | |
| 1009.36 | 683.07 | 106463.39 | |
| 750.85 | 614.79 | 18512.81 | |
| 167.51 | 174.74 | 52.28 | |
| 1080.12 | 708.09 | 138400.58 | |
| 1098.06 | 716.07 | 145912.41 | |
| 443.50 | 459.60 | 259.45 | |
| 82.99 | 131.07 | 2311.83 | |
| 173.11 | 259.55 | 7472.33 | |
| 140.73 | 137.23 | 12.22 | |
| 527.84 | 512.97 | 221.07 | |
| 925.54 | 674.20 | 63169.46 | |
| 310.66 | 365.39 | 2995.36 | |
| 686.90 | 559.45 | 16241.71 | |
| 173.45 | 194.15 | 428.41 | |
| 149.62 | 220.09 | 4966.25 | |

Average
RMSE: 146.03

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.63 | 1.81 | 7.10 | 3.99 | 150.85 |
| 9.18 | 3.04 | 2.75 | 4.23 | 893.78 |
| 3.06 | 1.72 | 5.41 | 3.84 | 121.32 |
| 9.17 | 2.28 | 5.21 | 4.09 | 893.12 |
| 5.90 | 2.74 | 5.91 | 3.89 | 322.75 |
| 5.94 | 2.92 | 5.02 | 4.02 | 316.19 |
| 1.82 | 2.28 | 3.04 | 4.21 | 101.58 |
| 6.33 | 2.00 | 4.49 | 3.71 | 360.77 |
| 7.02 | 2.87 | 5.05 | 4.12 | 468.41 |
| 2.38 | 2.80 | 6.35 | 3.77 | 138.34 |
| 5.35 | 2.05 | 6.37 | 4.23 | 283.76 |
| 1.22 | 2.12 | 7.94 | 4.05 | 152.47 |
| 5.90 | 1.93 | 6.22 | 3.91 | 343.19 |
| 3.37 | 2.36 | 6.35 | 3.88 | 159.26 |
| 9.21 | 2.62 | 6.20 | 4.13 | 917.37 |
| 5.59 | 2.05 | 5.69 | 4.33 | 304.20 |
| 5.69 | 2.05 | 2.27 | 3.94 | 271.61 |
| 2.68 | 1.58 | 7.28 | 4.03 | 141.27 |
| 4.96 | 3.10 | 4.02 | 4.09 | 230.74 |
| 7.48 | 4.07 | 4.41 | 3.97 | 526.45 |
| 5.62 | 2.13 | 2.21 | 3.95 | 269.32 |
| 1.05 | 3.42 | 7.46 | 4.23 | 138.90 |
| 4.47 | 3.30 | 7.35 | 3.83 | 248.68 |
| 6.92 | 2.31 | 7.01 | 4.22 | 483.61 |
| 6.92 | 2.99 | 5.15 | 4.04 | 466.84 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 5.13 | 2.44 | 3.46 | 3.84 | 246.42 |
| 8.77 | 3.44 | 6.53 | 4.07 | 841.77 |
| 8.27 | 3.16 | 7.77 | 4.05 | 724.34 |
| 4.40 | 3.31 | 6.71 | 4.16 | 219.63 |
| 1.43 | 1.45 | 3.72 | 4.13 | 111.58 |
| 5.96 | 2.43 | 3.68 | 4.31 | 336.86 |
| 3.20 | 2.21 | 2.79 | 3.67 | 127.84 |
| 8.39 | 2.22 | 7.84 | 4.14 | 742.67 |
| 3.64 | 3.14 | 4.96 | 4.02 | 164.56 |
| 9.21 | 2.62 | 6.20 | 4.13 | 917.37 |
| 5.57 | 2.89 | 6.41 | 4.19 | 311.78 |
| 6.81 | 2.22 | 7.11 | 4.00 | 469.58 |
| 6.22 | 3.81 | 6.85 | 4.25 | 404.60 |
| 7.17 | 2.93 | 5.61 | 4.04 | 505.13 |
| 4.09 | 2.69 | 2.36 | 4.37 | 160.19 |
| 5.76 | 3.10 | 7.63 | 3.67 | 349.75 |
| 4.66 | 2.12 | 4.57 | 3.76 | 222.58 |
| 8.79 | 2.34 | 2.60 | 3.94 | 775.37 |
| 4.95 | 2.19 | 7.52 | 3.91 | 259.22 |
| 2.69 | 3.63 | 5.60 | 4.15 | 144.00 |
| 8.01 | 3.04 | 7.51 | 4.17 | 686.90 |
| 2.72 | 2.31 | 7.64 | 3.64 | 154.07 |
| 3.29 | 3.10 | 4.34 | 3.81 | 149.75 |
| 6.33 | 2.00 | 4.49 | 3.71 | 360.77 |
| 5.64 | 2.04 | 6.58 | 4.11 | 338.63 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 9.98 | 1.62 | 4.55 | 3.72 | 1088.51 |
| 1.08 | 2.38 | 7.05 | 3.95 | 130.26 |
| 3.54 | 2.41 | 5.33 | 3.86 | 175.59 |
| 4.13 | 3.62 | 3.08 | 4.16 | 180.22 |
| 4.63 | 2.88 | 4.35 | 4.21 | 206.65 |
| 5.64 | 2.04 | 6.58 | 4.11 | 338.63 |
| 8.72 | 3.36 | 7.87 | 3.99 | 836.10 |
| 1.34 | 1.95 | 6.24 | 4.45 | 117.52 |
| 7.83 | 3.46 | 6.63 | 3.77 | 644.06 |
| 4.41 | 2.40 | 5.68 | 3.96 | 224.79 |
| 1.88 | 2.97 | 3.54 | 4.10 | 98.27 |
| 4.09 | 3.48 | 6.12 | 3.69 | 185.78 |
| 7.23 | 2.69 | 4.25 | 3.98 | 515.15 |
| 3.68 | 2.51 | 6.68 | 4.01 | 182.91 |
| 9.34 | 2.41 | 3.80 | 4.06 | 925.54 |
| 2.81 | 2.05 | 7.65 | 3.96 | 174.24 |
| 6.81 | 3.41 | 2.19 | 3.92 | 423.27 |
| 9.97 | 2.67 | 3.03 | 3.68 | 1101.66 |
| 7.13 | 3.26 | 5.23 | 4.11 | 494.21 |
| 8.57 | 2.74 | 4.38 | 3.87 | 750.31 |
| 6.67 | 2.06 | 3.08 | 3.82 | 414.27 |
| 2.20 | 3.63 | 2.06 | 4.20 | 105.80 |
| 4.09 | 2.69 | 2.36 | 4.37 | 160.19 |
| 4.85 | 2.70 | 2.50 | 4.09 | 217.86 |
| 6.92 | 2.99 | 5.15 | 4.04 | 466.84 |

**Model a**

| **Desired** | **Actual** | **SE** | **RMSE** |
|---|---|---|---|
| 1088.51 | 451.37 | 405955.28 | 233.77 |
| 130.26 | 77.81 | 2750.77 | |
| 175.59 | 204.24 | 820.79 | |
| 180.22 | 299.13 | 14140.64 | |
| 206.65 | 343.29 | 18670.15 | |
| 338.63 | 378.12 | 1559.58 | |
| 836.10 | 503.58 | 110568.19 | |
| 117.52 | 137.17 | 386.18 | |
| 644.06 | 379.27 | 70108.96 | |
| 224.79 | 270.48 | 2088.04 | |
| 98.27 | 142.89 | 1991.77 | |
| 185.78 | 200.94 | 230.02 | |
| 515.15 | 423.09 | 8475.05 | |
| 182.91 | 239.05 | 3151.66 | |
| 925.54 | 563.86 | 130806.33 | |
| 174.24 | 181.31 | 49.97 | |
| 423.27 | 381.23 | 1766.82 | |
| 1101.66 | 436.91 | 441892.94 | |
| 494.21 | 463.24 | 959.06 | |
| 750.31 | 447.96 | 91415.29 | |
| 414.27 | 345.47 | 4732.87 | |
| 105.80 | 177.81 | 5184.90 | |
| 160.19 | 347.80 | 35196.48 | |
| 217.86 | 326.26 | 11751.37 | |
| 466.84 | 426.87 | 1597.51 | |

**Model b**

| **Desired** | **Actual** | **SE** | **RMSE** |
|---|---|---|---|
| 1088.51 | 768.66 | 102307.21 | 120.67 |
| 130.26 | 39.18 | 8294.80 | |
| 175.59 | 191.02 | 238.13 | |
| 180.22 | 235.63 | 3070.23 | |
| 206.65 | 274.44 | 4595.45 | |
| 338.63 | 357.61 | 360.50 | |
| 836.10 | 641.27 | 37955.87 | |
| 117.52 | 51.78 | 4321.96 | |
| 644.06 | 554.94 | 7941.41 | |
| 224.79 | 257.00 | 1037.49 | |
| 98.27 | 81.96 | 266.00 | |
| 185.78 | 232.15 | 2150.66 | |
| 515.15 | 498.72 | 270.10 | |
| 182.91 | 201.52 | 346.32 | |
| 925.54 | 703.14 | 49461.16 | |
| 174.24 | 140.29 | 1153.07 | |
| 423.27 | 460.27 | 1369.10 | |
| 1101.66 | 767.57 | 111618.09 | |
| 494.21 | 489.47 | 22.45 | |
| 750.31 | 625.89 | 15479.70 | |
| 414.27 | 447.84 | 1127.11 | |
| 105.80 | 101.42 | 19.19 | |
| 160.19 | 231.91 | 5142.88 | |
| 217.86 | 291.63 | 5443.05 | |
| 466.84 | 470.38 | 12.51 | |

**Average**
**RMSE:** 177.22

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 6.36 | 3.12 | 7.46 | 3.98 | 405.95 |
| 5.35 | 2.05 | 6.37 | 4.23 | 283.76 |
| 9.02 | 2.31 | 5.47 | 4.44 | 887.32 |
| 3.39 | 3.12 | 5.69 | 3.77 | 165.89 |
| 9.36 | 2.78 | 2.72 | 3.90 | 923.60 |
| 4.04 | 2.58 | 7.67 | 3.99 | 215.84 |
| 1.72 | 3.02 | 7.11 | 4.02 | 137.50 |
| 9.16 | 2.58 | 6.61 | 4.13 | 905.39 |
| 7.30 | 1.78 | 3.01 | 3.95 | 489.89 |
| 6.34 | 2.69 | 6.51 | 4.05 | 400.19 |
| 9.44 | 2.98 | 5.74 | 3.89 | 979.19 |
| 4.63 | 2.88 | 4.35 | 4.21 | 208.65 |
| 6.76 | 2.85 | 4.47 | 3.85 | 427.85 |
| 4.51 | 3.24 | 4.58 | 4.37 | 214.65 |
| 8.34 | 2.83 | 4.17 | 4.11 | 703.42 |
| 4.26 | 2.58 | 6.54 | 4.05 | 205.11 |
| 3.42 | 2.56 | 5.49 | 3.95 | 168.57 |
| 4.13 | 3.22 | 4.98 | 4.19 | 185.69 |
| 5.80 | 3.19 | 6.18 | 4.28 | 320.89 |
| 7.78 | 3.07 | 5.92 | 4.17 | 621.37 |
| 3.35 | 2.92 | 7.55 | 4.14 | 173.45 |
| 4.05 | 2.40 | 6.34 | 3.84 | 192.66 |
| 2.50 | 2.64 | 6.03 | 4.15 | 140.27 |
| 7.21 | 3.20 | 3.49 | 3.95 | 489.68 |
| 7.44 | 2.71 | 6.85 | 3.79 | 540.47 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.53 | 2.72 | 3.60 | 4.22 | 109.24 |
| 9.19 | 2.09 | 3.92 | 3.71 | 896.59 |
| 4.05 | 2.40 | 6.34 | 3.84 | 192.66 |
| 8.18 | 2.85 | 6.03 | 4.25 | 694.60 |
| 2.53 | 2.84 | 3.62 | 3.86 | 118.78 |
| 9.02 | 2.31 | 5.47 | 4.44 | 887.32 |
| 9.65 | 2.82 | 4.57 | 3.89 | 1006.50 |
| 5.78 | 3.07 | 3.38 | 4.11 | 298.74 |
| 4.85 | 2.70 | 2.50 | 4.09 | 217.86 |
| 7.23 | 3.77 | 5.40 | 3.92 | 495.80 |
| 5.94 | 2.92 | 5.02 | 4.02 | 316.19 |
| 6.78 | 2.87 | 4.83 | 4.02 | 421.41 |
| 1.82 | 2.68 | 7.52 | 4.08 | 137.45 |
| 7.75 | 2.32 | 7.12 | 4.21 | 604.77 |
| 9.73 | 3.09 | 7.87 | 3.78 | 1080.12 |
| 1.03 | 2.71 | 7.57 | 4.01 | 139.51 |
| 6.77 | 2.84 | 3.58 | 3.91 | 399.06 |
| 4.41 | 2.40 | 5.68 | 3.96 | 224.79 |
| 8.02 | 2.54 | 5.01 | 3.85 | 640.83 |
| 2.77 | 2.52 | 2.47 | 4.00 | 113.58 |
| 7.35 | 2.74 | 5.48 | 3.85 | 517.29 |
| 4.26 | 2.58 | 6.54 | 4.05 | 205.11 |
| 5.00 | 2.42 | 5.69 | 3.80 | 245.03 |
| 3.64 | 3.30 | 3.46 | 4.08 | 171.26 |
| 9.93 | 2.72 | 3.22 | 3.84 | 1094.41 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 5.13 | 2.97 | 2.25 | 3.97 | 232.56 |
| 1.10 | 3.11 | 2.59 | 4.17 | 103.75 |
| 1.37 | 3.59 | 5.80 | 4.26 | 128.38 |
| 6.18 | 3.80 | 3.96 | 3.94 | 335.30 |
| 6.81 | 3.41 | 2.19 | 3.92 | 423.27 |
| 8.32 | 2.25 | 3.83 | 3.69 | 669.13 |
| 7.50 | 2.50 | 7.27 | 3.76 | 567.57 |
| 3.54 | 2.41 | 5.33 | 3.86 | 175.59 |
| 6.22 | 2.14 | 5.46 | 4.10 | 349.88 |
| 8.66 | 2.03 | 3.39 | 4.01 | 750.85 |
| 4.39 | 3.24 | 4.71 | 4.01 | 217.17 |
| 4.23 | 3.47 | 7.52 | 3.89 | 234.37 |
| 9.40 | 3.26 | 3.11 | 4.22 | 952.67 |
| 7.79 | 2.13 | 5.45 | 3.80 | 596.56 |
| 6.35 | 2.25 | 4.61 | 4.38 | 392.80 |
| 9.25 | 2.52 | 7.57 | 3.90 | 962.33 |
| 7.32 | 3.10 | 6.50 | 3.67 | 532.57 |
| 8.59 | 2.94 | 4.56 | 3.94 | 771.73 |
| 1.72 | 3.02 | 7.11 | 4.02 | 137.50 |
| 5.91 | 2.78 | 4.61 | 3.90 | 347.85 |
| 2.58 | 3.73 | 4.04 | 3.80 | 132.49 |
| 5.86 | 2.86 | 4.45 | 3.80 | 311.17 |
| 8.77 | 3.44 | 6.53 | 4.07 | 841.77 |
| 4.06 | 3.89 | 5.30 | 3.90 | 182.68 |
| 5.00 | 2.42 | 5.69 | 3.80 | 245.03 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 232.56 | 336.37 | 10776.45 | 107.04 |
| 103.75 | -125.75 | 52673.25 | |
| 128.38 | -94.46 | 49657.02 | |
| 335.30 | 455.66 | 14485.78 | |
| 423.27 | 528.29 | 11030.40 | |
| 669.13 | 701.33 | 1036.90 | |
| 567.57 | 607.29 | 1577.91 | |
| 175.59 | 153.26 | 498.65 | |
| 349.88 | 460.76 | 12293.70 | |
| 750.85 | 739.74 | 123.51 | |
| 217.17 | 251.56 | 1183.11 | |
| 234.37 | 233.08 | 1.66 | |
| 952.67 | 824.39 | 16454.90 | |
| 596.56 | 640.10 | 1896.13 | |
| 392.80 | 475.77 | 6884.29 | |
| 962.33 | 807.64 | 23928.55 | |
| 532.57 | 586.97 | 2960.06 | |
| 771.73 | 732.10 | 1570.19 | |
| 137.50 | -55.07 | 37083.74 | |
| 347.85 | 425.41 | 6016.15 | |
| 132.49 | 43.73 | 7877.96 | |
| 311.17 | 419.65 | 11768.79 | |
| 841.77 | 752.35 | 7994.62 | |
| 182.68 | 212.80 | 907.19 | |
| 245.03 | 320.88 | 5753.38 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 232.56 | 365.52 | 17678.32 | 106.36 |
| 103.75 | -82.83 | 34813.14 | |
| 128.38 | -52.47 | 32705.82 | |
| 335.30 | 481.25 | 21301.90 | |
| 423.27 | 551.72 | 16500.89 | |
| 669.13 | 719.60 | 2547.59 | |
| 567.57 | 628.37 | 3696.45 | |
| 175.59 | 187.86 | 150.73 | |
| 349.88 | 486.20 | 18583.09 | |
| 750.85 | 756.87 | 36.17 | |
| 217.17 | 283.24 | 4365.80 | |
| 234.37 | 265.31 | 957.29 | |
| 952.67 | 839.00 | 12921.28 | |
| 596.56 | 660.20 | 4050.32 | |
| 392.80 | 500.77 | 11656.87 | |
| 962.33 | 822.74 | 19483.79 | |
| 532.57 | 608.65 | 5789.31 | |
| 771.73 | 749.46 | 495.96 | |
| 137.50 | -14.25 | 23029.37 | |
| 347.85 | 451.91 | 10828.52 | |
| 132.49 | 81.60 | 2589.33 | |
| 311.17 | 446.32 | 18266.09 | |
| 841.77 | 769.11 | 5279.58 | |
| 182.68 | 245.64 | 3963.08 | |
| 245.03 | 350.49 | 11122.46 | |

Average
RMSE: 106.70

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 5.65 | 1.92 | 7.79 | 3.87 | 329.69 |
| 9.08 | 2.48 | 6.53 | 4.00 | 864.17 |
| 2.72 | 2.85 | 3.26 | 4.25 | 122.93 |
| 3.44 | 2.82 | 3.44 | 4.14 | 151.33 |
| 6.92 | 2.99 | 5.15 | 4.04 | 466.84 |
| 5.35 | 2.05 | 6.37 | 4.23 | 283.76 |
| 2.26 | 2.93 | 6.61 | 3.83 | 146.87 |
| 1.77 | 2.45 | 2.18 | 3.91 | 76.79 |
| 5.13 | 2.44 | 3.46 | 3.84 | 246.42 |
| 7.97 | 3.02 | 6.57 | 3.99 | 658.66 |
| 9.80 | 2.22 | 2.79 | 4.27 | 1070.64 |
| 5.42 | 2.91 | 2.12 | 4.17 | 263.19 |
| 5.99 | 1.96 | 6.13 | 3.66 | 326.85 |
| 2.40 | 2.06 | 2.59 | 4.43 | 126.01 |
| 8.51 | 2.60 | 6.10 | 3.86 | 735.82 |
| 1.03 | 4.30 | 5.87 | 3.98 | 97.50 |
| 9.93 | 2.72 | 3.22 | 3.84 | 1094.41 |
| 9.58 | 2.61 | 2.89 | 3.50 | 977.06 |
| 1.35 | 2.50 | 4.74 | 3.91 | 102.81 |
| 1.80 | 3.60 | 7.93 | 4.06 | 161.77 |
| 5.65 | 2.10 | 5.47 | 3.74 | 311.22 |
| 5.51 | 3.00 | 2.38 | 4.08 | 281.92 |
| 1.87 | 1.86 | 3.57 | 3.96 | 101.99 |
| 5.93 | 2.62 | 5.23 | 3.75 | 343.92 |
| 2.70 | 3.47 | 6.47 | 4.19 | 171.87 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 1.20 | 2.63 | 2.91 | 3.91 | 99.59 |
| 7.63 | 2.70 | 3.42 | 4.10 | 564.34 |
| 6.43 | 3.24 | 6.32 | 4.16 | 392.85 |
| 6.76 | 2.85 | 4.47 | 3.85 | 427.85 |
| 5.29 | 2.40 | 6.28 | 4.12 | 289.47 |
| 9.43 | 3.06 | 5.20 | 3.94 | 974.81 |
| 9.36 | 2.78 | 2.72 | 3.90 | 923.60 |
| 6.53 | 2.94 | 7.44 | 3.79 | 419.52 |
| 3.06 | 1.72 | 5.41 | 3.84 | 121.32 |
| 8.19 | 3.21 | 2.35 | 3.93 | 640.93 |
| 2.20 | 3.63 | 2.06 | 4.20 | 105.80 |
| 2.96 | 3.38 | 5.88 | 4.21 | 147.88 |
| 7.78 | 3.07 | 5.92 | 4.17 | 621.37 |
| 3.24 | 2.70 | 3.29 | 3.89 | 120.70 |
| 2.27 | 3.46 | 2.07 | 4.31 | 102.08 |
| 6.38 | 2.42 | 3.72 | 4.43 | 367.83 |
| 2.71 | 2.97 | 4.59 | 3.76 | 120.52 |
| 4.54 | 3.42 | 4.22 | 4.14 | 207.60 |
| 5.35 | 2.05 | 6.37 | 4.23 | 283.76 |
| 2.53 | 2.84 | 3.62 | 3.86 | 118.78 |
| 9.34 | 2.41 | 3.80 | 4.06 | 925.54 |
| 2.77 | 2.52 | 2.47 | 4.00 | 113.58 |
| 3.38 | 3.44 | 3.07 | 3.63 | 123.79 |
| 4.20 | 2.94 | 2.72 | 4.22 | 168.33 |
| 2.81 | 2.05 | 7.65 | 3.86 | 174.24 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 6.24 | 2.50 | 7.89 | 4.16 | 413.48 |
| 5.59 | 2.05 | 5.69 | 4.33 | 304.20 |
| 4.09 | 2.69 | 2.36 | 4.37 | 160.19 |
| 9.73 | 3.09 | 7.87 | 3.78 | 1080.12 |
| 3.87 | 2.45 | 4.23 | 4.11 | 172.26 |
| 6.84 | 2.61 | 4.66 | 4.09 | 427.61 |
| 6.69 | 2.10 | 4.66 | 3.76 | 429.47 |
| 8.10 | 3.33 | 3.27 | 4.17 | 656.30 |
| 4.11 | 2.95 | 3.90 | 4.30 | 181.20 |
| 3.66 | 2.61 | 3.81 | 3.99 | 150.07 |
| 8.72 | 3.36 | 7.87 | 3.99 | 836.10 |
| 3.35 | 2.92 | 7.55 | 4.14 | 173.45 |
| 7.07 | 2.70 | 5.37 | 4.05 | 488.96 |
| 7.41 | 2.77 | 3.64 | 4.31 | 563.17 |
| 4.41 | 2.40 | 5.68 | 3.96 | 224.79 |
| 6.67 | 2.06 | 3.08 | 3.82 | 414.27 |
| 9.63 | 3.24 | 7.78 | 4.21 | 1073.54 |
| 1.76 | 2.84 | 4.68 | 4.11 | 116.72 |
| 8.12 | 3.12 | 2.24 | 4.10 | 646.90 |
| 2.04 | 3.43 | 6.67 | 4.12 | 126.93 |
| 9.25 | 2.52 | 7.57 | 3.90 | 962.33 |
| 3.68 | 2.51 | 6.68 | 4.01 | 182.91 |
| 1.69 | 2.50 | 7.91 | 3.62 | 144.89 |
| 1.77 | 2.45 | 2.18 | 3.91 | 76.79 |
| 3.42 | 2.56 | 5.49 | 3.95 | 168.57 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 413.48 | 428.71 | 232.19 | 51.55 |
| 304.20 | 352.43 | 2326.63 | |
| 160.19 | 207.86 | 2271.84 | |
| 1080.12 | 980.89 | 9845.47 | |
| 172.26 | 190.84 | 345.31 | |
| 427.61 | 506.91 | 6288.97 | |
| 429.47 | 486.54 | 3257.60 | |
| 656.30 | 693.57 | 1388.94 | |
| 181.20 | 209.71 | 812.80 | |
| 150.07 | 175.31 | 637.20 | |
| 836.10 | 796.88 | 1537.98 | |
| 173.45 | 153.48 | 399.08 | |
| 488.96 | 538.68 | 2472.33 | |
| 563.17 | 586.42 | 540.79 | |
| 224.79 | 235.27 | 109.78 | |
| 414.27 | 484.04 | 4868.64 | |
| 1073.54 | 963.00 | 12220.14 | |
| 116.72 | 72.90 | 1920.21 | |
| 646.90 | 695.59 | 2370.89 | |
| 126.93 | 83.50 | 1886.11 | |
| 962.33 | 891.19 | 5060.21 | |
| 182.91 | 176.58 | 40.15 | |
| 144.89 | 70.77 | 5493.88 | |
| 76.79 | 73.34 | 11.95 | |
| 168.57 | 158.28 | 105.83 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 413.48 | 411.54 | 3.75 | 55.17 |
| 304.20 | 336.73 | 1058.27 | |
| 160.19 | 194.94 | 1207.15 | |
| 1080.12 | 953.09 | 16136.93 | |
| 172.26 | 178.25 | 35.88 | |
| 427.61 | 488.23 | 3675.27 | |
| 429.47 | 468.25 | 1504.56 | |
| 656.30 | 671.30 | 224.85 | |
| 181.20 | 196.75 | 241.93 | |
| 150.07 | 163.02 | 167.69 | |
| 836.10 | 772.62 | 4029.75 | |
| 173.45 | 141.60 | 1014.40 | |
| 488.96 | 519.39 | 926.09 | |
| 563.17 | 566.21 | 9.27 | |
| 224.79 | 221.82 | 8.82 | |
| 414.27 | 465.80 | 2656.07 | |
| 1073.54 | 935.54 | 19045.94 | |
| 116.72 | 62.58 | 2931.26 | |
| 646.90 | 673.28 | 695.86 | |
| 126.93 | 72.97 | 2911.07 | |
| 962.33 | 865.11 | 9450.67 | |
| 182.91 | 164.26 | 347.96 | |
| 144.89 | 60.49 | 7123.46 | |
| 76.79 | 63.01 | 190.04 | |
| 168.57 | 146.32 | 495.19 | |

Average
RMSE: 53.36

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 6.39 | 1.83 | 5.54 | 3.76 | 379.46 |
| 6.09 | 2.80 | 2.54 | 3.94 | 324.80 |
| 7.75 | 2.32 | 7.12 | 4.21 | 604.77 |
| 7.48 | 4.07 | 4.41 | 3.97 | 526.45 |
| 9.26 | 2.25 | 4.46 | 4.32 | 918.17 |
| 4.26 | 2.58 | 6.54 | 4.05 | 205.11 |
| 4.13 | 2.76 | 6.72 | 3.95 | 187.61 |
| 7.13 | 3.26 | 5.23 | 4.11 | 494.21 |
| 4.06 | 3.89 | 5.30 | 3.90 | 182.68 |
| 1.34 | 1.95 | 6.24 | 4.45 | 117.52 |
| 6.53 | 2.94 | 7.44 | 3.79 | 419.52 |
| 9.08 | 2.48 | 6.53 | 4.00 | 864.17 |
| 3.87 | 2.82 | 7.71 | 3.59 | 200.84 |
| 8.09 | 3.55 | 5.27 | 3.79 | 656.89 |
| 6.88 | 3.69 | 6.42 | 4.11 | 499.42 |
| 6.67 | 2.06 | 3.08 | 3.82 | 414.27 |
| 6.01 | 1.91 | 2.78 | 3.64 | 315.36 |
| 6.86 | 2.48 | 7.98 | 3.90 | 485.61 |
| 4.41 | 2.40 | 5.68 | 3.96 | 224.79 |
| 6.18 | 3.80 | 3.96 | 3.94 | 335.30 |
| 9.36 | 2.78 | 2.72 | 3.90 | 923.60 |
| 2.43 | 2.48 | 7.42 | 3.77 | 147.10 |
| 9.13 | 2.33 | 2.08 | 4.09 | 860.20 |
| 6.76 | 2.85 | 4.47 | 3.85 | 427.85 |
| 8.79 | 2.34 | 2.60 | 3.94 | 775.37 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 7.63 | 2.70 | 3.42 | 4.10 | 564.34 |
| 9.58 | 2.47 | 5.33 | 3.63 | 1002.07 |
| 3.20 | 2.21 | 2.79 | 3.67 | 127.84 |
| 6.81 | 2.36 | 2.02 | 3.98 | 429.73 |
| 4.11 | 2.95 | 3.90 | 4.30 | 181.20 |
| 8.62 | 2.39 | 3.45 | 4.51 | 760.18 |
| 5.29 | 2.40 | 6.28 | 4.12 | 289.47 |
| 9.58 | 2.61 | 2.89 | 3.50 | 977.06 |
| 4.16 | 3.51 | 4.58 | 4.26 | 179.11 |
| 2.67 | 3.16 | 5.41 | 4.32 | 157.98 |
| 9.65 | 2.82 | 4.57 | 3.89 | 1006.50 |
| 1.77 | 2.45 | 2.18 | 3.91 | 76.79 |
| 3.29 | 3.10 | 4.34 | 3.81 | 149.75 |
| 8.84 | 2.01 | 4.07 | 4.00 | 811.16 |
| 1.56 | 3.42 | 6.67 | 4.17 | 107.76 |
| 4.18 | 3.26 | 6.52 | 3.66 | 218.27 |
| 7.75 | 2.32 | 7.12 | 4.21 | 604.77 |
| 1.82 | 2.28 | 3.04 | 4.21 | 101.58 |
| 7.07 | 2.58 | 6.43 | 3.90 | 484.24 |
| 3.39 | 3.12 | 5.69 | 3.77 | 165.89 |
| 6.48 | 3.75 | 2.15 | 4.09 | 370.25 |
| 1.54 | 2.50 | 5.73 | 4.00 | 135.15 |
| 7.23 | 3.77 | 5.40 | 3.92 | 495.80 |
| 2.47 | 1.95 | 3.92 | 3.75 | 116.08 |
| 2.09 | 3.12 | 3.66 | 4.19 | 121.92 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.86 | 2.95 | 4.85 | 4.04 | 821.76 |
| 3.37 | 2.36 | 6.35 | 3.88 | 159.26 |
| 5.59 | 2.05 | 5.69 | 4.33 | 304.20 |
| 5.16 | 2.36 | 3.34 | 4.18 | 242.60 |
| 7.69 | 2.70 | 7.55 | 4.03 | 629.76 |
| 9.58 | 2.47 | 5.33 | 3.63 | 1002.07 |
| 5.76 | 3.10 | 7.63 | 3.67 | 349.75 |
| 5.53 | 2.89 | 5.05 | 3.78 | 288.47 |
| 7.80 | 3.20 | 7.46 | 3.92 | 610.80 |
| 3.81 | 4.02 | 6.65 | 4.07 | 187.34 |
| 8.14 | 2.86 | 4.94 | 4.19 | 674.04 |
| 5.89 | 2.80 | 7.61 | 4.02 | 356.75 |
| 2.36 | 2.62 | 2.93 | 4.16 | 130.54 |
| 7.47 | 2.57 | 3.13 | 4.23 | 529.17 |
| 8.40 | 2.54 | 5.21 | 3.52 | 736.81 |
| 4.45 | 3.66 | 2.74 | 4.03 | 198.89 |
| 6.54 | 2.76 | 6.43 | 4.24 | 433.44 |
| 8.67 | 3.89 | 4.65 | 3.87 | 773.67 |
| 5.99 | 1.96 | 6.13 | 3.66 | 326.85 |
| 4.13 | 3.62 | 3.08 | 4.16 | 180.22 |
| 7.30 | 1.78 | 3.01 | 3.95 | 489.89 |
| 5.81 | 2.42 | 4.31 | 3.63 | 321.51 |
| 1.54 | 3.12 | 3.70 | 4.00 | 97.47 |
| 3.30 | 2.70 | 2.11 | 4.21 | 132.71 |
| 7.52 | 2.27 | 4.23 | 3.86 | 535.36 |

**Model a**

| **Desired** | **Actual** | **SE** | **RMSE** |
|---|---|---|---|
| 821.76 | 813.70 | 64.95 | 12.10 |
| 159.26 | 167.54 | 68.63 | |
| 304.20 | 298.04 | 37.96 | |
| 242.60 | 241.69 | 0.83 | |
| 629.76 | 601.28 | 811.16 | |
| 1002.07 | 1002.58 | 0.27 | |
| 349.75 | 337.54 | 149.29 | |
| 288.47 | 286.42 | 4.22 | |
| 610.80 | 620.61 | 96.19 | |
| 187.34 | 188.10 | 0.58 | |
| 674.04 | 658.79 | 232.58 | |
| 356.75 | 349.65 | 50.50 | |
| 130.54 | 114.27 | 264.57 | |
| 529.17 | 521.70 | 55.83 | |
| 736.81 | 713.27 | 554.22 | |
| 198.89 | 188.85 | 100.77 | |
| 433.44 | 411.20 | 494.69 | |
| 773.67 | 769.09 | 20.99 | |
| 326.85 | 342.83 | 255.38 | |
| 180.22 | 173.09 | 50.80 | |
| 489.89 | 493.33 | 11.80 | |
| 321.51 | 307.89 | 185.43 | |
| 97.47 | 109.28 | 139.57 | |
| 132.71 | 133.79 | 1.18 | |
| 535.36 | 538.26 | 8.44 | |

**Model b**

| **Desired** | **Actual** | **SE** | **RMSE** |
|---|---|---|---|
| 821.76 | 814.21 | 57.00 | 11.79 |
| 159.26 | 168.67 | 88.66 | |
| 304.20 | 298.88 | 28.34 | |
| 242.60 | 241.74 | 0.74 | |
| 629.76 | 603.03 | 714.50 | |
| 1002.07 | 1003.27 | 1.45 | |
| 349.75 | 339.33 | 108.64 | |
| 288.47 | 287.00 | 2.16 | |
| 610.80 | 622.31 | 132.36 | |
| 187.34 | 189.38 | 4.15 | |
| 674.04 | 659.33 | 216.39 | |
| 356.75 | 351.42 | 28.37 | |
| 130.54 | 114.23 | 266.02 | |
| 529.17 | 521.70 | 55.81 | |
| 736.81 | 713.91 | 524.27 | |
| 198.89 | 188.77 | 102.46 | |
| 433.44 | 412.37 | 444.10 | |
| 773.67 | 769.53 | 17.17 | |
| 326.85 | 343.86 | 289.30 | |
| 180.22 | 173.08 | 50.93 | |
| 489.89 | 493.30 | 11.63 | |
| 321.51 | 308.22 | 176.61 | |
| 97.47 | 109.43 | 143.04 | |
| 132.71 | 133.60 | 0.79 | |
| 535.36 | 538.56 | 10.28 | |

**Average**
**RMSE:** 11.95

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 3.87 | 3.68 | 2.16 | 3.87 | 151.90 |
| 7.19 | 3.42 | 2.06 | 3.85 | 469.72 |
| 8.49 | 2.33 | 5.89 | 3.98 | 755.43 |
| 6.22 | 3.11 | 7.27 | 4.14 | 391.53 |
| 9.26 | 2.25 | 4.46 | 4.32 | 918.17 |
| 5.29 | 2.40 | 6.28 | 4.12 | 289.47 |
| 4.08 | 2.88 | 2.43 | 4.13 | 162.23 |
| 1.88 | 3.33 | 2.09 | 3.82 | 85.69 |
| 3.06 | 1.72 | 5.41 | 3.84 | 121.32 |
| 2.50 | 2.75 | 3.60 | 4.25 | 118.98 |
| 4.69 | 2.17 | 2.64 | 3.90 | 211.47 |
| 4.71 | 3.00 | 5.60 | 4.04 | 215.92 |
| 3.43 | 3.49 | 2.68 | 3.78 | 147.47 |
| 7.63 | 2.07 | 4.49 | 4.21 | 560.03 |
| 4.95 | 2.93 | 2.03 | 3.90 | 227.50 |
| 1.25 | 3.32 | 2.40 | 4.15 | 111.27 |
| 4.16 | 3.51 | 4.58 | 4.26 | 179.11 |
| 2.04 | 3.43 | 6.67 | 4.12 | 126.93 |
| 6.34 | 2.69 | 6.51 | 4.05 | 400.19 |
| 1.99 | 2.72 | 2.87 | 3.76 | 101.76 |
| 6.68 | 2.84 | 7.80 | 4.07 | 470.40 |
| 8.51 | 2.60 | 6.10 | 3.86 | 735.82 |
| 8.22 | 2.40 | 7.91 | 3.92 | 720.06 |
| 1.38 | 2.35 | 2.05 | 3.99 | 90.93 |
| 6.84 | 2.61 | 4.66 | 4.09 | 427.61 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 6.31 | 3.43 | 3.22 | 4.04 | 361.47 |
| 8.91 | 3.88 | 5.30 | 4.14 | 867.33 |
| 3.64 | 3.14 | 4.96 | 4.02 | 164.56 |
| 7.32 | 3.18 | 3.53 | 3.78 | 514.00 |
| 9.04 | 2.76 | 3.89 | 3.61 | 842.89 |
| 6.35 | 2.25 | 4.61 | 4.38 | 392.60 |
| 6.88 | 3.69 | 6.42 | 4.11 | 499.42 |
| 4.23 | 3.47 | 7.52 | 3.89 | 234.37 |
| 7.17 | 3.63 | 3.83 | 4.19 | 507.21 |
| 6.43 | 3.24 | 6.32 | 4.16 | 392.85 |
| 6.78 | 2.87 | 4.83 | 4.02 | 421.41 |
| 6.18 | 2.24 | 5.31 | 3.53 | 349.35 |
| 7.01 | 2.81 | 5.07 | 3.91 | 468.90 |
| 8.98 | 3.20 | 2.23 | 4.15 | 845.34 |
| 1.87 | 1.86 | 3.57 | 3.96 | 101.99 |
| 7.13 | 3.26 | 5.23 | 4.11 | 494.21 |
| 6.96 | 2.32 | 5.01 | 4.15 | 471.56 |
| 1.23 | 2.93 | 3.59 | 4.04 | 92.39 |
| 5.35 | 2.05 | 6.37 | 4.23 | 283.76 |
| 3.87 | 3.07 | 6.87 | 4.13 | 196.77 |
| 8.09 | 3.55 | 5.27 | 3.79 | 656.89 |
| 7.21 | 3.20 | 3.49 | 3.95 | 489.68 |
| 5.80 | 3.19 | 6.18 | 4.28 | 320.89 |
| 6.63 | 2.61 | 3.02 | 4.00 | 401.24 |
| 4.14 | 3.24 | 6.01 | 4.00 | 204.42 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 6.86 | 2.48 | 7.98 | 3.90 | 485.61 |
| 3.38 | 3.44 | 3.07 | 3.63 | 123.79 |
| 4.11 | 2.48 | 3.10 | 3.95 | 152.17 |
| 3.82 | 3.55 | 5.03 | 4.23 | 199.07 |
| 4.74 | 2.77 | 7.84 | 3.81 | 237.71 |
| 5.35 | 2.05 | 6.37 | 4.23 | 283.76 |
| 6.92 | 2.31 | 7.01 | 4.22 | 483.61 |
| 1.43 | 1.45 | 3.72 | 4.13 | 111.58 |
| 4.23 | 3.47 | 7.52 | 3.89 | 234.37 |
| 2.59 | 2.54 | 2.57 | 4.08 | 107.74 |
| 8.99 | 2.87 | 3.86 | 4.43 | 859.51 |
| 3.29 | 3.10 | 4.34 | 3.81 | 149.75 |
| 7.25 | 2.12 | 4.41 | 4.11 | 485.37 |
| 4.06 | 3.84 | 7.44 | 3.84 | 208.11 |
| 3.98 | 2.46 | 4.45 | 3.72 | 151.11 |
| 2.67 | 3.16 | 5.41 | 4.32 | 157.98 |
| 6.63 | 2.81 | 3.02 | 4.00 | 401.24 |
| 1.26 | 3.04 | 4.15 | 3.77 | 101.87 |
| 9.93 | 2.72 | 3.22 | 3.84 | 1094.41 |
| 6.68 | 2.84 | 7.80 | 4.07 | 470.40 |
| 3.43 | 3.49 | 2.68 | 3.78 | 147.47 |
| 8.22 | 3.24 | 5.99 | 4.15 | 699.75 |
| 8.86 | 2.95 | 4.85 | 4.04 | 821.76 |
| 9.21 | 2.62 | 6.20 | 4.13 | 917.37 |
| 4.69 | 2.73 | 2.26 | 4.00 | 199.51 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 485.61 | 480.41 | 27.03 | 30.22 |
| 123.79 | 151.32 | 757.90 | |
| 152.17 | 187.38 | 1239.75 | |
| 199.07 | 182.55 | 273.15 | |
| 237.71 | 258.77 | 443.41 | |
| 283.76 | 296.03 | 150.48 | |
| 483.61 | 475.02 | 73.85 | |
| 111.58 | 87.90 | 560.85 | |
| 234.37 | 221.64 | 162.16 | |
| 107.74 | 118.67 | 119.49 | |
| 859.51 | 785.61 | 5461.97 | |
| 149.75 | 153.15 | 11.59 | |
| 485.37 | 482.72 | 7.03 | |
| 208.11 | 210.11 | 3.99 | |
| 151.11 | 187.65 | 1335.11 | |
| 157.98 | 132.45 | 651.67 | |
| 401.24 | 386.99 | 203.06 | |
| 101.87 | 84.76 | 292.83 | |
| 1094.41 | 1008.86 | 7318.60 | |
| 470.40 | 453.57 | 283.23 | |
| 147.47 | 151.91 | 19.78 | |
| 699.75 | 670.42 | 859.95 | |
| 821.76 | 778.07 | 1909.02 | |
| 917.37 | 897.49 | 394.86 | |
| 199.51 | 215.93 | 269.52 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 485.61 | 441.70 | 1928.26 | 28.27 |
| 123.79 | 168.73 | 2020.17 | |
| 152.17 | 200.14 | 2301.27 | |
| 199.07 | 192.71 | 40.48 | |
| 237.71 | 242.45 | 22.43 | |
| 283.76 | 280.88 | 8.28 | |
| 483.61 | 446.51 | 1377.08 | |
| 111.58 | 88.42 | 536.45 | |
| 234.37 | 216.18 | 330.98 | |
| 107.74 | 129.32 | 465.87 | |
| 859.51 | 829.89 | 877.36 | |
| 149.75 | 162.00 | 150.05 | |
| 485.37 | 486.45 | 1.17 | |
| 208.11 | 208.77 | 0.43 | |
| 151.11 | 192.30 | 1696.59 | |
| 157.98 | 136.06 | 480.59 | |
| 401.24 | 419.22 | 323.08 | |
| 101.87 | 90.10 | 138.52 | |
| 1094.41 | 1078.67 | 247.64 | |
| 470.40 | 425.97 | 1973.33 | |
| 147.47 | 171.80 | 592.24 | |
| 699.75 | 675.52 | 587.13 | |
| 821.76 | 800.83 | 438.21 | |
| 917.37 | 873.92 | 1887.37 | |
| 199.51 | 239.02 | 1560.91 | |

Average
RMSE: 29.25

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 5.24 | 2.84 | 5.12 | 4.05 | 274.98 |
| 6.77 | 3.16 | 5.13 | 4.05 | 447.51 |
| 4.56 | 3.58 | 3.13 | 3.58 | 196.64 |
| 9.25 | 2.52 | 7.57 | 3.90 | 962.33 |
| 7.32 | 3.18 | 3.53 | 3.78 | 514.00 |
| 9.44 | 2.70 | 7.94 | 3.74 | 1009.36 |
| 4.69 | 2.73 | 2.26 | 4.00 | 199.51 |
| 4.25 | 2.44 | 3.52 | 3.83 | 173.11 |
| 5.85 | 2.62 | 3.62 | 3.88 | 328.05 |
| 7.75 | 2.32 | 7.12 | 4.21 | 604.77 |
| 1.64 | 3.36 | 3.18 | 3.98 | 103.04 |
| 3.80 | 3.05 | 2.47 | 3.97 | 166.03 |
| 6.09 | 2.80 | 2.54 | 3.94 | 324.80 |
| 8.71 | 2.65 | 3.60 | 3.76 | 755.05 |
| 9.43 | 3.06 | 5.20 | 3.94 | 974.81 |
| 2.26 | 3.22 | 2.91 | 4.06 | 97.67 |
| 3.37 | 3.92 | 2.79 | 3.97 | 138.06 |
| 5.16 | 3.00 | 2.37 | 3.98 | 243.10 |
| 8.08 | 3.38 | 2.91 | 3.94 | 636.97 |
| 8.62 | 2.39 | 3.45 | 4.51 | 760.18 |
| 7.92 | 2.24 | 3.47 | 3.65 | 602.92 |
| 4.96 | 2.80 | 3.63 | 4.23 | 217.27 |
| 7.30 | 1.78 | 3.01 | 3.95 | 489.89 |
| 5.64 | 3.85 | 4.06 | 3.89 | 295.97 |
| 2.58 | 2.40 | 3.15 | 3.77 | 86.91 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 2.51 | 3.25 | 3.76 | 4.01 | 140.73 |
| 5.53 | 2.73 | 7.84 | 3.65 | 314.22 |
| 6.12 | 3.15 | 5.02 | 4.02 | 371.94 |
| 1.89 | 2.73 | 5.97 | 4.05 | 124.24 |
| 9.50 | 2.85 | 6.98 | 3.88 | 1012.81 |
| 1.88 | 2.97 | 3.54 | 4.10 | 98.27 |
| 8.32 | 3.42 | 4.42 | 4.26 | 696.90 |
| 1.26 | 2.08 | 6.06 | 3.81 | 127.72 |
| 6.36 | 3.12 | 7.46 | 3.98 | 405.95 |
| 1.92 | 3.59 | 3.46 | 4.04 | 102.97 |
| 8.76 | 3.30 | 7.47 | 4.31 | 842.59 |
| 7.32 | 3.18 | 3.53 | 3.78 | 514.00 |
| 7.23 | 2.69 | 4.25 | 3.98 | 515.15 |
| 4.69 | 3.32 | 5.68 | 3.99 | 227.61 |
| 2.43 | 2.48 | 7.42 | 3.77 | 147.10 |
| 7.32 | 3.10 | 6.50 | 3.67 | 532.57 |
| 9.69 | 2.94 | 3.13 | 3.98 | 1022.72 |
| 6.33 | 2.00 | 4.49 | 3.71 | 360.77 |
| 6.48 | 3.75 | 2.15 | 4.09 | 370.25 |
| 6.92 | 2.31 | 7.01 | 4.22 | 483.61 |
| 4.41 | 2.40 | 5.68 | 3.96 | 224.79 |
| 4.96 | 3.10 | 4.02 | 4.09 | 230.74 |
| 4.17 | 1.75 | 4.63 | 3.78 | 196.33 |
| 5.32 | 2.94 | 4.26 | 4.11 | 252.90 |
| 3.64 | 3.30 | 3.46 | 4.08 | 171.26 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 2.69 | 3.63 | 5.60 | 4.15 | 144.00 |
| 4.41 | 2.40 | 5.68 | 3.96 | 224.79 |
| 7.17 | 2.93 | 5.61 | 4.04 | 505.13 |
| 3.87 | 2.82 | 7.71 | 3.59 | 200.84 |
| 7.44 | 2.71 | 6.85 | 3.79 | 540.47 |
| 6.44 | 2.58 | 7.46 | 3.71 | 413.33 |
| 3.64 | 3.30 | 3.46 | 4.08 | 171.26 |
| 3.80 | 3.05 | 2.47 | 3.97 | 166.03 |
| 5.96 | 2.43 | 3.68 | 4.31 | 336.86 |
| 1.02 | 2.28 | 2.60 | 4.15 | 86.61 |
| 1.37 | 3.59 | 5.80 | 4.26 | 128.38 |
| 4.16 | 3.51 | 4.58 | 4.26 | 179.11 |
| 1.82 | 2.28 | 3.04 | 4.21 | 101.58 |
| 2.70 | 3.47 | 6.47 | 4.19 | 171.87 |
| 7.02 | 2.62 | 7.56 | 4.09 | 488.64 |
| 2.23 | 2.65 | 4.42 | 3.97 | 109.83 |
| 2.09 | 3.02 | 4.92 | 4.11 | 107.54 |
| 2.38 | 2.80 | 6.35 | 3.77 | 138.34 |
| 3.66 | 2.77 | 7.70 | 3.99 | 198.50 |
| 1.81 | 2.45 | 5.29 | 3.99 | 104.23 |
| 7.38 | 2.92 | 3.04 | 3.93 | 514.12 |
| 5.98 | 2.93 | 2.55 | 4.06 | 314.65 |
| 1.03 | 2.71 | 7.57 | 4.01 | 139.51 |
| 2.63 | 2.52 | 2.66 | 4.08 | 111.85 |
| 1.64 | 2.59 | 5.65 | 3.96 | 137.57 |

**Model a**

| **Desired** | **Actual** | **SE** | **RMSE** |
|---|---|---|---|
| 144.00 | 138.45 | 30.82 | 55.43 |
| 224.79 | 280.02 | 3051.06 | |
| 505.13 | 554.92 | 2479.32 | |
| 200.84 | 258.86 | 3365.96 | |
| 540.47 | 626.45 | 7393.62 | |
| 413.33 | 526.26 | 12754.13 | |
| 171.26 | 179.35 | 65.41 | |
| 166.03 | 169.87 | 14.73 | |
| 336.86 | 369.23 | 1047.94 | |
| 86.61 | 26.94 | 3560.48 | |
| 128.38 | 53.96 | 5538.87 | |
| 179.11 | 239.44 | 3639.73 | |
| 101.58 | 64.48 | 1376.28 | |
| 171.87 | 146.00 | 669.25 | |
| 488.64 | 596.91 | 11723.81 | |
| 109.83 | 97.61 | 149.48 | |
| 107.54 | 92.76 | 218.36 | |
| 138.34 | 121.87 | 271.19 | |
| 198.50 | 238.71 | 1616.82 | |
| 104.23 | 77.53 | 712.71 | |
| 514.12 | 467.18 | 2203.18 | |
| 314.65 | 326.01 | 128.90 | |
| 139.51 | 39.64 | 9975.23 | |
| 111.85 | 103.76 | 65.44 | |
| 137.57 | 68.66 | 4748.99 | |

**Model b**

| **Desired** | **Actual** | **SE** | **RMSE** |
|---|---|---|---|
| 144.00 | 158.47 | 209.23 | 43.55 |
| 224.79 | 278.32 | 2865.95 | |
| 505.13 | 484.27 | 435.17 | |
| 200.84 | 239.88 | 1523.94 | |
| 540.47 | 504.98 | 1259.03 | |
| 413.33 | 428.49 | 229.94 | |
| 171.26 | 223.28 | 2705.68 | |
| 166.03 | 234.83 | 4733.99 | |
| 336.86 | 392.55 | 3101.27 | |
| 86.61 | 52.55 | 1160.47 | |
| 128.38 | 73.48 | 3014.24 | |
| 179.11 | 260.56 | 6635.12 | |
| 101.58 | 101.63 | 0.00 | |
| 171.87 | 158.85 | 169.51 | |
| 488.64 | 472.46 | 261.57 | |
| 109.83 | 127.79 | 322.31 | |
| 107.54 | 119.04 | 132.38 | |
| 138.34 | 138.05 | 0.09 | |
| 198.50 | 224.79 | 691.04 | |
| 104.23 | 100.93 | 10.92 | |
| 514.12 | 500.14 | 195.33 | |
| 314.65 | 393.48 | 6213.17 | |
| 139.51 | 53.22 | 7446.89 | |
| 111.85 | 154.59 | 1826.94 | |
| 137.57 | 89.86 | 2276.34 | |

**Average**
**RMSE:** 49.49

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 7.17 | 2.93 | 5.61 | 4.04 | 505.13 |
| 1.61 | 4.14 | 4.67 | 3.84 | 98.04 |
| 9.53 | 2.20 | 4.82 | 4.22 | 978.95 |
| 6.69 | 2.10 | 4.66 | 3.76 | 429.47 |
| 5.64 | 2.04 | 6.58 | 4.11 | 338.63 |
| 8.12 | 3.12 | 2.24 | 4.10 | 646.90 |
| 2.40 | 2.89 | 3.99 | 3.74 | 106.42 |
| 3.47 | 2.21 | 7.44 | 4.13 | 175.75 |
| 4.83 | 2.50 | 4.65 | 4.19 | 232.12 |
| 7.35 | 2.76 | 4.69 | 3.95 | 510.36 |
| 3.37 | 2.36 | 6.35 | 3.88 | 159.26 |
| 5.56 | 2.34 | 5.59 | 4.11 | 295.21 |
| 1.82 | 2.28 | 3.04 | 4.21 | 101.58 |
| 6.95 | 3.17 | 7.54 | 3.98 | 489.65 |
| 4.56 | 3.58 | 3.13 | 3.58 | 196.64 |
| 7.21 | 3.20 | 3.49 | 3.95 | 489.68 |
| 1.54 | 3.12 | 3.70 | 4.00 | 97.47 |
| 5.71 | 2.10 | 3.77 | 4.30 | 300.71 |
| 1.37 | 3.59 | 5.80 | 4.26 | 128.38 |
| 2.64 | 2.41 | 5.60 | 3.78 | 141.42 |
| 9.21 | 2.62 | 6.20 | 4.13 | 917.37 |
| 7.92 | 2.87 | 7.65 | 3.75 | 652.28 |
| 5.13 | 2.97 | 2.25 | 3.97 | 232.56 |
| 9.38 | 2.87 | 2.23 | 4.15 | 932.80 |
| 1.03 | 4.30 | 5.87 | 3.98 | 97.50 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 4.93 | 3.19 | 7.27 | 4.07 | 275.68 |
| 9.80 | 2.22 | 2.79 | 4.27 | 1070.64 |
| 6.77 | 3.16 | 5.13 | 4.05 | 447.51 |
| 3.72 | 2.77 | 2.05 | 3.92 | 133.14 |
| 9.42 | 3.31 | 5.79 | 4.04 | 967.23 |
| 6.69 | 2.10 | 4.66 | 3.76 | 429.47 |
| 3.38 | 2.66 | 3.89 | 4.01 | 136.89 |
| 7.60 | 3.21 | 5.00 | 4.10 | 585.09 |
| 2.38 | 3.10 | 7.54 | 4.04 | 168.96 |
| 5.96 | 2.43 | 3.68 | 4.31 | 336.86 |
| 2.23 | 2.65 | 4.42 | 3.97 | 109.83 |
| 4.39 | 3.24 | 4.71 | 4.01 | 217.17 |
| 5.42 | 2.91 | 2.12 | 4.17 | 263.19 |
| 7.53 | 3.02 | 6.41 | 4.18 | 569.89 |
| 5.29 | 2.40 | 6.28 | 4.12 | 289.47 |
| 4.40 | 3.31 | 6.71 | 4.16 | 219.63 |
| 4.74 | 2.77 | 7.84 | 3.81 | 237.71 |
| 3.47 | 2.21 | 7.44 | 4.13 | 175.75 |
| 6.53 | 2.94 | 7.44 | 3.79 | 419.52 |
| 4.11 | 2.48 | 3.10 | 3.95 | 152.17 |
| 8.59 | 2.50 | 5.54 | 3.68 | 779.17 |
| 1.63 | 1.81 | 7.10 | 3.99 | 150.85 |
| 1.38 | 2.94 | 2.64 | 4.24 | 107.69 |
| 4.83 | 2.50 | 4.65 | 4.19 | 232.12 |
| 1.88 | 3.33 | 2.09 | 3.82 | 85.69 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 4.69 | 2.73 | 2.26 | 4.00 | 199.51 |
| 7.97 | 3.02 | 6.57 | 3.99 | 658.66 |
| 8.03 | 2.39 | 5.80 | 3.98 | 655.40 |
| 1.01 | 2.43 | 6.08 | 4.18 | 133.72 |
| 9.09 | 3.84 | 2.34 | 3.85 | 865.06 |
| 7.41 | 2.27 | 5.85 | 3.97 | 538.15 |
| 2.42 | 2.83 | 2.67 | 3.69 | 82.99 |
| 7.38 | 2.92 | 3.04 | 3.93 | 514.12 |
| 5.65 | 1.92 | 7.79 | 3.87 | 329.69 |
| 2.72 | 2.74 | 5.81 | 3.69 | 143.91 |
| 8.27 | 3.16 | 7.77 | 4.05 | 724.34 |
| 6.25 | 2.47 | 7.18 | 4.00 | 402.72 |
| 9.34 | 2.41 | 3.80 | 4.06 | 925.54 |
| 7.83 | 3.46 | 6.63 | 3.77 | 644.06 |
| 5.56 | 2.34 | 5.59 | 4.11 | 295.21 |
| 7.69 | 2.70 | 7.55 | 4.03 | 629.76 |
| 7.01 | 2.81 | 5.07 | 3.91 | 468.90 |
| 9.26 | 2.25 | 4.46 | 4.32 | 918.17 |
| 3.06 | 1.72 | 5.41 | 3.84 | 121.32 |
| 2.71 | 2.30 | 7.80 | 4.02 | 155.12 |
| 4.16 | 3.51 | 4.58 | 4.26 | 179.11 |
| 7.32 | 3.10 | 6.50 | 3.67 | 532.57 |
| 9.37 | 2.33 | 5.91 | 4.09 | 960.29 |
| 5.85 | 2.62 | 3.62 | 3.88 | 328.05 |
| 7.13 | 3.26 | 5.23 | 4.11 | 494.21 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 199.51 | 320.90 | 14735.66 | 92.38 |
| 658.66 | 634.04 | 606.25 | |
| 655.40 | 639.10 | 265.60 | |
| 133.72 | -30.24 | 26884.56 | |
| 865.06 | 740.07 | 15623.20 | |
| 538.15 | 579.92 | 1744.48 | |
| 82.99 | 104.31 | 454.46 | |
| 514.12 | 576.95 | 3947.56 | |
| 329.69 | 412.73 | 6896.25 | |
| 143.91 | 132.67 | 126.15 | |
| 724.34 | 662.17 | 3864.72 | |
| 402.72 | 469.57 | 4468.93 | |
| 925.54 | 764.62 | 25895.46 | |
| 644.06 | 620.39 | 559.85 | |
| 295.21 | 403.98 | 11830.70 | |
| 629.76 | 606.84 | 525.21 | |
| 468.90 | 542.10 | 5357.97 | |
| 918.17 | 757.04 | 25961.58 | |
| 121.32 | 165.41 | 1944.22 | |
| 155.12 | 132.30 | 520.59 | |
| 179.11 | 270.44 | 8341.72 | |
| 532.57 | 572.03 | 1557.21 | |
| 960.29 | 767.68 | 37097.04 | |
| 328.05 | 431.01 | 10600.18 | |
| 494.21 | 553.63 | 3530.62 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 199.51 | 301.73 | 10448.51 | 88.41 |
| 658.66 | 643.57 | 227.68 | |
| 655.40 | 649.10 | 39.69 | |
| 133.72 | -81.61 | 46365.88 | |
| 865.06 | 759.32 | 11180.96 | |
| 538.15 | 584.49 | 2147.26 | |
| 82.99 | 65.28 | 313.63 | |
| 514.12 | 581.24 | 4506.25 | |
| 329.69 | 401.97 | 5225.69 | |
| 143.91 | 96.25 | 2271.42 | |
| 724.34 | 674.29 | 2505.49 | |
| 402.72 | 464.03 | 3758.47 | |
| 925.54 | 786.12 | 19437.32 | |
| 644.06 | 628.68 | 236.52 | |
| 295.21 | 392.43 | 9450.27 | |
| 629.76 | 613.88 | 252.11 | |
| 468.90 | 543.21 | 5520.93 | |
| 918.17 | 777.85 | 19688.97 | |
| 121.32 | 131.98 | 113.78 | |
| 155.12 | 95.84 | 3513.88 | |
| 179.11 | 246.64 | 4560.91 | |
| 532.57 | 575.88 | 1875.74 | |
| 960.29 | 789.47 | 29160.12 | |
| 328.05 | 421.93 | 8813.10 | |
| 494.21 | 555.79 | 3792.16 | |

| Average | |
|---|---|
| RMSE: | 90.40 |

MLR Model 11.xls

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 6.44 | 2.58 | 7.46 | 3.71 | 413.33 |
| 9.73 | 3.09 | 7.87 | 3.78 | 1080.12 |
| 5.16 | 2.36 | 3.34 | 4.18 | 242.60 |
| 3.24 | 2.70 | 3.29 | 3.89 | 120.70 |
| 7.02 | 2.87 | 5.05 | 4.12 | 468.41 |
| 1.81 | 2.45 | 5.29 | 3.99 | 104.23 |
| 8.98 | 3.20 | 2.23 | 4.15 | 845.34 |
| 1.59 | 2.64 | 7.05 | 4.19 | 154.13 |
| 2.71 | 2.30 | 7.80 | 4.02 | 155.12 |
| 7.35 | 2.74 | 5.48 | 3.85 | 517.29 |
| 6.22 | 3.11 | 7.27 | 4.14 | 391.53 |
| 2.09 | 3.02 | 4.92 | 4.11 | 107.54 |
| 7.80 | 3.20 | 7.46 | 3.92 | 610.80 |
| 6.69 | 2.46 | 3.20 | 4.13 | 424.55 |
| 4.11 | 2.48 | 3.10 | 3.95 | 152.17 |
| 4.16 | 3.51 | 4.58 | 4.26 | 179.11 |
| 4.83 | 2.50 | 4.65 | 4.19 | 232.12 |
| 5.76 | 3.10 | 7.63 | 3.67 | 349.75 |
| 2.81 | 3.00 | 2.76 | 3.85 | 133.15 |
| 8.96 | 2.93 | 6.40 | 3.99 | 862.69 |
| 3.06 | 1.72 | 5.41 | 3.84 | 121.32 |
| 7.17 | 2.93 | 5.61 | 4.04 | 505.13 |
| 8.51 | 2.60 | 6.10 | 3.86 | 735.82 |
| 6.25 | 2.47 | 7.18 | 4.00 | 402.72 |
| 8.66 | 2.03 | 3.39 | 4.01 | 750.85 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 3.43 | 3.49 | 2.68 | 3.78 | 147.47 |
| 7.76 | 2.58 | 7.95 | 3.99 | 619.75 |
| 9.98 | 1.62 | 4.55 | 3.72 | 1088.51 |
| 8.79 | 3.26 | 6.04 | 3.89 | 818.72 |
| 6.17 | 3.73 | 4.48 | 4.28 | 365.36 |
| 9.53 | 2.20 | 4.82 | 4.22 | 978.95 |
| 9.31 | 2.44 | 4.86 | 3.70 | 916.60 |
| 2.58 | 2.40 | 3.15 | 3.77 | 86.91 |
| 8.13 | 2.95 | 5.61 | 3.87 | 676.06 |
| 1.02 | 2.28 | 2.60 | 4.15 | 86.61 |
| 7.23 | 3.77 | 5.40 | 3.92 | 495.80 |
| 8.12 | 3.05 | 7.96 | 4.07 | 714.80 |
| 9.36 | 2.78 | 2.72 | 3.90 | 923.60 |
| 6.34 | 2.69 | 6.51 | 4.05 | 400.19 |
| 7.79 | 2.13 | 5.45 | 3.80 | 596.56 |
| 3.44 | 2.82 | 3.44 | 4.14 | 151.33 |
| 1.20 | 2.63 | 2.91 | 3.91 | 99.59 |
| 8.84 | 3.03 | 4.97 | 4.06 | 817.71 |
| 4.03 | 2.67 | 4.25 | 4.59 | 198.81 |
| 5.94 | 2.92 | 5.02 | 4.02 | 316.19 |
| 6.09 | 2.80 | 2.54 | 3.94 | 324.80 |
| 2.36 | 2.62 | 2.93 | 4.16 | 130.54 |
| 6.76 | 3.08 | 7.54 | 4.40 | 475.07 |
| 6.96 | 2.32 | 5.01 | 4.15 | 471.56 |
| 2.09 | 3.12 | 3.66 | 4.19 | 121.92 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.77 | 4.23 | 4.43 | 4.30 | 819.25 |
| 2.40 | 2.06 | 2.59 | 4.43 | 126.01 |
| 6.16 | 2.56 | 6.60 | 3.95 | 388.12 |
| 5.51 | 3.00 | 2.38 | 4.08 | 281.92 |
| 9.04 | 2.76 | 3.89 | 3.61 | 842.89 |
| 8.49 | 2.91 | 6.83 | 3.98 | 747.72 |
| 5.85 | 2.62 | 3.62 | 3.88 | 328.05 |
| 4.53 | 2.69 | 2.73 | 4.17 | 189.20 |
| 4.63 | 2.88 | 4.35 | 4.21 | 206.65 |
| 6.49 | 3.15 | 2.72 | 3.68 | 360.99 |
| 3.14 | 2.63 | 3.95 | 4.36 | 139.79 |
| 6.32 | 3.01 | 3.41 | 3.96 | 373.29 |
| 4.87 | 2.61 | 4.51 | 3.65 | 219.97 |
| 7.41 | 2.77 | 3.64 | 4.31 | 563.17 |
| 8.56 | 4.07 | 3.63 | 4.24 | 772.36 |
| 6.84 | 2.61 | 4.66 | 4.09 | 427.61 |
| 6.76 | 2.85 | 4.47 | 3.85 | 427.85 |
| 2.27 | 3.46 | 2.07 | 4.31 | 102.08 |
| 9.93 | 2.72 | 3.22 | 3.84 | 1094.41 |
| 7.13 | 3.26 | 5.23 | 4.11 | 494.21 |
| 8.77 | 3.44 | 6.53 | 4.07 | 841.77 |
| 1.70 | 2.86 | 4.19 | 4.13 | 91.37 |
| 2.50 | 2.64 | 6.03 | 4.15 | 140.27 |
| 9.37 | 2.33 | 5.91 | 4.09 | 960.29 |
| 8.66 | 2.03 | 3.39 | 4.01 | 750.85 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 819.25 | 774.00 | 2048.19 | 44.34 |
| 126.01 | 64.80 | 3746.57 | |
| 388.12 | 415.68 | 759.85 | |
| 281.92 | 303.70 | 474.69 | |
| 842.89 | 812.52 | 922.48 | |
| 747.72 | 752.83 | 26.10 | |
| 328.05 | 347.59 | 381.81 | |
| 189.20 | 209.34 | 405.53 | |
| 206.65 | 230.71 | 578.76 | |
| 360.99 | 419.54 | 3428.00 | |
| 139.79 | 114.71 | 628.87 | |
| 373.29 | 403.24 | 896.87 | |
| 219.97 | 250.96 | 960.85 | |
| 563.17 | 551.30 | 140.84 | |
| 772.36 | 730.71 | 1735.16 | |
| 427.61 | 481.05 | 2855.83 | |
| 427.85 | 467.18 | 1546.82 | |
| 102.08 | 55.86 | 2136.40 | |
| 1094.41 | 973.52 | 14614.06 | |
| 494.21 | 526.22 | 1024.73 | |
| 841.77 | 796.10 | 2085.17 | |
| 91.37 | 47.26 | 1945.05 | |
| 140.27 | 100.09 | 1614.54 | |
| 960.29 | 895.74 | 4166.34 | |
| 750.85 | 744.98 | 34.46 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 819.25 | 798.60 | 426.42 | 44.77 |
| 126.01 | 87.07 | 1516.04 | |
| 388.12 | 409.16 | 442.63 | |
| 281.92 | 333.30 | 2640.53 | |
| 842.89 | 845.79 | 8.41 | |
| 747.72 | 750.64 | 8.56 | |
| 328.05 | 371.02 | 1846.63 | |
| 189.20 | 234.68 | 2068.35 | |
| 206.65 | 243.87 | 1384.69 | |
| 360.99 | 450.77 | 8061.23 | |
| 139.79 | 128.21 | 134.02 | |
| 373.29 | 429.18 | 3123.31 | |
| 219.97 | 265.99 | 2118.44 | |
| 563.17 | 577.87 | 216.22 | |
| 772.36 | 761.73 | 113.13 | |
| 427.61 | 497.71 | 4914.47 | |
| 427.85 | 486.64 | 3455.44 | |
| 102.08 | 80.88 | 449.33 | |
| 1094.41 | 1014.43 | 6396.98 | |
| 494.21 | 537.79 | 1899.06 | |
| 841.77 | 798.12 | 1905.31 | |
| 91.37 | 58.07 | 1108.54 | |
| 140.27 | 91.67 | 2361.80 | |
| 960.29 | 908.07 | 2726.99 | |
| 750.85 | 778.91 | 787.48 | |

Average
RMSE: 44.56

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 2.56 | 3.17 | 7.54 | 4.15 | 155.10 |
| 3.49 | 2.94 | 4.18 | 3.95 | 152.06 |
| 4.09 | 3.48 | 6.12 | 3.69 | 185.78 |
| 4.52 | 3.28 | 4.83 | 4.19 | 218.05 |
| 1.22 | 2.12 | 7.94 | 4.05 | 152.47 |
| 4.53 | 2.69 | 2.73 | 4.17 | 189.20 |
| 9.18 | 3.04 | 2.75 | 4.23 | 893.78 |
| 4.11 | 2.95 | 3.90 | 4.30 | 181.20 |
| 4.18 | 3.26 | 6.52 | 3.66 | 218.27 |
| 5.16 | 3.00 | 2.37 | 3.98 | 243.10 |
| 3.93 | 2.58 | 3.12 | 3.79 | 173.29 |
| 6.92 | 2.31 | 7.01 | 4.22 | 483.61 |
| 6.92 | 2.99 | 5.15 | 4.04 | 466.84 |
| 9.04 | 2.62 | 6.14 | 4.22 | 902.75 |
| 2.63 | 2.52 | 2.66 | 4.08 | 111.85 |
| 2.62 | 2.80 | 4.72 | 4.13 | 113.92 |
| 4.25 | 2.44 | 3.52 | 3.83 | 173.11 |
| 5.26 | 2.88 | 3.34 | 4.14 | 262.00 |
| 7.65 | 2.96 | 4.69 | 4.07 | 550.36 |
| 7.41 | 2.27 | 5.85 | 3.97 | 538.15 |
| 7.77 | 3.14 | 2.04 | 4.05 | 574.13 |
| 8.12 | 3.05 | 7.96 | 4.07 | 714.80 |
| 8.40 | 2.54 | 5.21 | 3.52 | 736.81 |
| 6.36 | 3.12 | 7.46 | 3.98 | 405.95 |
| 7.63 | 2.07 | 4.49 | 4.21 | 560.03 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 4.13 | 3.62 | 3.08 | 4.16 | 180.22 |
| 6.86 | 2.48 | 7.98 | 3.90 | 485.61 |
| 5.76 | 3.10 | 7.63 | 3.67 | 349.75 |
| 9.94 | 2.85 | 3.83 | 4.03 | 1107.42 |
| 2.39 | 2.21 | 5.46 | 3.60 | 121.95 |
| 7.46 | 3.17 | 3.10 | 3.98 | 527.84 |
| 4.17 | 1.75 | 4.63 | 3.78 | 196.33 |
| 5.94 | 3.88 | 4.27 | 3.69 | 322.75 |
| 2.64 | 2.41 | 5.60 | 3.78 | 141.42 |
| 1.76 | 2.84 | 4.68 | 4.11 | 116.72 |
| 6.68 | 2.84 | 7.80 | 4.07 | 470.40 |
| 1.53 | 2.72 | 3.60 | 4.22 | 109.24 |
| 6.19 | 3.02 | 4.69 | 3.99 | 357.59 |
| 7.23 | 2.69 | 4.25 | 3.98 | 515.15 |
| 6.63 | 2.81 | 3.02 | 4.00 | 401.24 |
| 9.63 | 3.24 | 7.78 | 4.21 | 1073.54 |
| 5.06 | 3.16 | 2.36 | 4.07 | 240.02 |
| 7.52 | 2.27 | 4.23 | 3.86 | 535.36 |
| 5.81 | 2.42 | 4.31 | 3.63 | 321.51 |
| 4.33 | 2.77 | 7.60 | 3.93 | 228.76 |
| 8.08 | 3.38 | 2.91 | 3.94 | 636.97 |
| 3.68 | 2.51 | 6.68 | 4.01 | 182.91 |
| 8.27 | 3.16 | 7.77 | 4.05 | 724.34 |
| 4.54 | 3.42 | 4.22 | 4.14 | 207.60 |
| 4.32 | 1.82 | 4.19 | 3.68 | 170.50 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.66 | 2.03 | 3.39 | 4.01 | 750.85 |
| 4.04 | 3.00 | 4.57 | 3.89 | 159.03 |
| 1.14 | 2.41 | 2.30 | 4.07 | 91.91 |
| 9.63 | 3.24 | 7.78 | 4.21 | 1073.54 |
| 5.93 | 4.10 | 4.77 | 3.75 | 326.03 |
| 3.81 | 2.39 | 6.20 | 3.68 | 185.03 |
| 3.38 | 2.66 | 3.89 | 4.01 | 136.89 |
| 6.33 | 2.00 | 4.49 | 3.71 | 360.77 |
| 9.16 | 2.58 | 6.61 | 4.13 | 905.39 |
| 1.43 | 1.45 | 3.72 | 4.13 | 111.58 |
| 2.77 | 2.52 | 2.47 | 4.00 | 113.58 |
| 6.18 | 3.80 | 3.96 | 3.94 | 335.30 |
| 1.37 | 2.72 | 4.43 | 4.18 | 120.06 |
| 8.59 | 2.94 | 4.56 | 3.94 | 771.73 |
| 3.87 | 3.07 | 6.87 | 4.13 | 196.77 |
| 3.91 | 3.48 | 5.76 | 3.56 | 172.30 |
| 2.72 | 2.74 | 5.81 | 3.69 | 143.91 |
| 7.92 | 2.87 | 7.65 | 3.75 | 652.28 |
| 7.02 | 2.87 | 5.05 | 4.12 | 468.41 |
| 5.97 | 2.21 | 3.96 | 4.03 | 319.85 |
| 5.19 | 2.46 | 4.78 | 3.62 | 246.75 |
| 8.34 | 2.83 | 4.17 | 4.11 | 703.42 |
| 6.35 | 2.25 | 4.61 | 4.38 | 392.80 |
| 8.84 | 2.01 | 4.07 | 4.00 | 811.16 |
| 8.57 | 2.74 | 4.38 | 3.87 | 750.31 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 750.85 | 772.73 | 478.74 | 9.97 |
| 159.03 | 176.96 | 321.34 | |
| 91.91 | 95.05 | 9.87 | |
| 1073.54 | 1079.21 | 32.16 | |
| 326.03 | 325.20 | 0.69 | |
| 185.03 | 180.95 | 16.66 | |
| 136.89 | 143.38 | 42.08 | |
| 360.77 | 369.33 | 73.21 | |
| 905.39 | 929.60 | 585.88 | |
| 111.58 | 105.44 | 37.75 | |
| 113.58 | 116.26 | 7.17 | |
| 335.30 | 347.95 | 159.90 | |
| 120.06 | 111.24 | 77.74 | |
| 771.73 | 766.28 | 29.64 | |
| 196.77 | 196.73 | 0.00 | |
| 172.30 | 179.79 | 56.08 | |
| 143.91 | 140.07 | 14.69 | |
| 652.28 | 657.55 | 27.73 | |
| 468.41 | 472.79 | 19.20 | |
| 319.85 | 324.47 | 21.36 | |
| 246.75 | 253.83 | 50.15 | |
| 703.42 | 706.49 | 9.44 | |
| 392.80 | 377.22 | 242.75 | |
| 811.16 | 821.41 | 105.02 | |
| 750.31 | 758.47 | 66.68 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 750.85 | 768.31 | 304.68 | 9.58 |
| 159.03 | 180.69 | 469.30 | |
| 91.91 | 99.69 | 60.58 | |
| 1073.54 | 1071.35 | 4.79 | |
| 326.03 | 326.95 | 0.84 | |
| 185.03 | 184.83 | 0.04 | |
| 136.89 | 147.50 | 112.54 | |
| 360.77 | 370.45 | 93.60 | |
| 905.39 | 923.51 | 328.15 | |
| 111.58 | 110.06 | 2.32 | |
| 113.58 | 120.63 | 49.58 | |
| 335.30 | 349.31 | 196.22 | |
| 120.06 | 115.86 | 17.56 | |
| 771.73 | 762.07 | 93.33 | |
| 196.77 | 200.58 | 14.54 | |
| 172.30 | 183.62 | 127.99 | |
| 143.91 | 144.45 | 0.30 | |
| 652.28 | 655.27 | 8.89 | |
| 468.41 | 472.61 | 17.68 | |
| 319.85 | 326.15 | 39.78 | |
| 246.75 | 256.53 | 95.76 | |
| 703.42 | 703.05 | 0.14 | |
| 392.80 | 378.29 | 210.46 | |
| 811.16 | 816.40 | 27.39 | |
| 750.31 | 754.34 | 16.23 | |

Average
RMSE: 9.77

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 4.16 | 3.51 | 4.58 | 4.26 | 179.11 |
| 4.69 | 2.17 | 2.64 | 3.90 | 211.47 |
| 5.65 | 2.10 | 5.47 | 3.74 | 311.22 |
| 4.26 | 2.58 | 6.54 | 4.05 | 205.11 |
| 3.64 | 3.30 | 3.46 | 4.08 | 171.26 |
| 6.18 | 3.80 | 3.96 | 3.94 | 335.30 |
| 9.63 | 3.24 | 7.78 | 4.21 | 1073.54 |
| 5.13 | 2.97 | 2.25 | 3.97 | 232.56 |
| 3.43 | 3.49 | 2.68 | 3.78 | 147.47 |
| 9.26 | 2.25 | 4.46 | 4.32 | 918.17 |
| 5.62 | 2.13 | 2.21 | 3.95 | 269.32 |
| 1.53 | 3.19 | 6.28 | 4.03 | 129.40 |
| 7.23 | 2.69 | 4.25 | 3.98 | 515.15 |
| 3.20 | 2.21 | 2.79 | 3.67 | 127.84 |
| 8.39 | 2.22 | 7.84 | 4.14 | 742.67 |
| 5.64 | 3.85 | 4.06 | 3.89 | 295.97 |
| 7.45 | 2.91 | 7.35 | 3.84 | 558.55 |
| 5.50 | 2.37 | 3.80 | 4.04 | 289.91 |
| 1.03 | 4.30 | 5.87 | 3.98 | 97.50 |
| 4.87 | 2.86 | 2.32 | 3.96 | 202.25 |
| 1.35 | 2.50 | 4.74 | 3.91 | 102.81 |
| 8.45 | 2.72 | 3.40 | 3.71 | 713.60 |
| 7.44 | 3.41 | 3.78 | 3.68 | 552.02 |
| 9.19 | 2.09 | 3.92 | 3.71 | 896.59 |
| 2.64 | 2.41 | 5.60 | 3.78 | 141.42 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 5.64 | 3.24 | 5.34 | 3.87 | 310.66 |
| 4.11 | 2.95 | 3.90 | 4.30 | 181.20 |
| 9.50 | 2.85 | 6.98 | 3.88 | 1012.81 |
| 6.25 | 2.47 | 7.18 | 4.00 | 402.72 |
| 6.54 | 2.76 | 6.43 | 4.24 | 433.44 |
| 3.66 | 2.77 | 7.70 | 3.99 | 198.50 |
| 5.85 | 3.31 | 3.00 | 4.02 | 311.22 |
| 6.38 | 2.42 | 3.72 | 4.43 | 367.83 |
| 3.80 | 3.05 | 2.47 | 3.97 | 166.03 |
| 7.44 | 2.71 | 6.85 | 3.79 | 540.47 |
| 6.23 | 3.15 | 6.72 | 3.41 | 369.31 |
| 2.50 | 2.75 | 3.60 | 4.25 | 118.98 |
| 2.59 | 2.54 | 2.57 | 4.08 | 107.74 |
| 5.56 | 2.34 | 5.59 | 4.11 | 295.21 |
| 7.38 | 2.92 | 3.04 | 3.93 | 514.12 |
| 8.90 | 2.79 | 2.94 | 3.96 | 812.38 |
| 9.65 | 2.82 | 4.57 | 3.89 | 1006.50 |
| 2.45 | 3.32 | 4.65 | 4.00 | 120.87 |
| 5.69 | 2.05 | 2.27 | 3.94 | 271.61 |
| 3.14 | 1.77 | 7.10 | 4.20 | 176.23 |
| 5.71 | 2.10 | 3.77 | 4.30 | 300.71 |
| 5.13 | 2.97 | 2.25 | 3.97 | 232.56 |
| 5.00 | 2.42 | 5.69 | 3.80 | 245.03 |
| 3.75 | 2.45 | 3.44 | 4.04 | 168.78 |
| 6.76 | 2.85 | 4.47 | 3.85 | 427.85 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 2.45 | 3.32 | 4.65 | 4.00 | 120.87 |
| 5.00 | 2.42 | 5.69 | 3.80 | 245.03 |
| 1.63 | 1.81 | 7.10 | 3.99 | 150.85 |
| 4.25 | 2.44 | 3.52 | 3.83 | 173.11 |
| 9.80 | 2.22 | 2.79 | 4.27 | 1070.64 |
| 7.38 | 2.92 | 3.04 | 3.93 | 514.12 |
| 5.29 | 2.40 | 6.28 | 4.12 | 289.47 |
| 3.38 | 2.66 | 3.89 | 4.01 | 136.89 |
| 9.43 | 3.06 | 5.20 | 3.94 | 974.81 |
| 2.70 | 3.47 | 6.47 | 4.19 | 171.87 |
| 7.02 | 2.87 | 5.05 | 4.12 | 468.41 |
| 5.13 | 2.97 | 2.25 | 3.97 | 232.56 |
| 8.51 | 2.60 | 6.10 | 3.86 | 735.82 |
| 4.74 | 2.77 | 7.84 | 3.81 | 237.71 |
| 7.30 | 1.78 | 3.01 | 3.95 | 489.89 |
| 9.34 | 2.41 | 3.80 | 4.06 | 925.54 |
| 9.37 | 2.33 | 5.91 | 4.09 | 960.29 |
| 4.40 | 3.31 | 6.71 | 4.16 | 219.63 |
| 3.27 | 2.48 | 6.41 | 4.10 | 154.92 |
| 4.13 | 2.76 | 6.72 | 3.95 | 187.61 |
| 8.34 | 2.83 | 4.17 | 4.11 | 703.42 |
| 2.04 | 3.43 | 6.67 | 4.12 | 126.93 |
| 3.01 | 3.02 | 7.88 | 4.01 | 181.21 |
| 9.13 | 2.33 | 2.08 | 4.09 | 860.20 |
| 1.80 | 3.60 | 7.93 | 4.06 | 161.77 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 120.87 | 127.30 | 41.38 | 50.99 |
| 245.03 | 274.48 | 867.39 | |
| 150.85 | 114.33 | 1333.46 | |
| 173.11 | 199.82 | 713.13 | |
| 1070.64 | 916.08 | 23888.39 | |
| 514.12 | 469.16 | 2020.89 | |
| 289.47 | 306.52 | 290.40 | |
| 136.89 | 159.26 | 500.35 | |
| 974.81 | 931.23 | 1899.56 | |
| 171.87 | 149.40 | 504.95 | |
| 468.41 | 468.78 | 0.14 | |
| 232.56 | 240.30 | 59.86 | |
| 735.82 | 750.48 | 215.03 | |
| 237.71 | 283.60 | 2105.45 | |
| 489.89 | 458.58 | 980.60 | |
| 925.54 | 846.62 | 6227.59 | |
| 960.29 | 948.73 | 133.69 | |
| 219.63 | 243.82 | 585.13 | |
| 154.92 | 175.02 | 404.06 | |
| 187.61 | 226.25 | 1492.54 | |
| 703.42 | 649.77 | 2878.87 | |
| 126.93 | 125.37 | 2.44 | |
| 181.21 | 175.18 | 36.42 | |
| 860.20 | 731.91 | 16457.77 | |
| 161.77 | 124.86 | 1361.74 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 120.87 | 118.12 | 7.57 | 33.79 |
| 245.03 | 259.42 | 207.26 | |
| 150.85 | 101.99 | 2387.03 | |
| 173.11 | 191.97 | 355.52 | |
| 1070.64 | 982.33 | 7798.15 | |
| 514.12 | 478.03 | 1302.19 | |
| 289.47 | 294.63 | 26.54 | |
| 136.89 | 151.25 | 206.08 | |
| 974.81 | 957.80 | 289.39 | |
| 171.87 | 138.18 | 1135.22 | |
| 468.41 | 468.72 | 0.10 | |
| 232.56 | 238.54 | 35.75 | |
| 735.82 | 749.18 | 178.54 | |
| 237.71 | 260.04 | 498.68 | |
| 489.89 | 467.11 | 519.24 | |
| 925.54 | 886.93 | 1490.73 | |
| 960.29 | 975.51 | 231.72 | |
| 219.63 | 230.75 | 123.49 | |
| 154.92 | 162.54 | 58.06 | |
| 187.61 | 210.27 | 513.19 | |
| 703.42 | 668.76 | 1201.42 | |
| 126.93 | 114.00 | 167.09 | |
| 181.21 | 158.69 | 507.37 | |
| 860.20 | 777.98 | 6759.63 | |
| 161.77 | 111.29 | 2547.40 | |

Average
RMSE 42.39

Training Sample 1

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 2.37 | 2.93 | 7.66 | 3.96 | 155.45 |
| 9.02 | 2.31 | 5.47 | 4.44 | 887.32 |
| 5.80 | 3.19 | 6.18 | 4.28 | 320.89 |
| 4.04 | 2.58 | 7.67 | 3.99 | 215.84 |
| 8.09 | 3.55 | 5.27 | 3.79 | 656.89 |
| 4.11 | 2.48 | 3.10 | 3.95 | 152.17 |
| 4.53 | 2.69 | 2.73 | 4.17 | 189.20 |
| 3.66 | 2.77 | 7.70 | 3.99 | 198.50 |
| 5.64 | 3.85 | 4.06 | 3.89 | 295.97 |
| 7.02 | 2.62 | 7.56 | 4.09 | 488.64 |
| 8.03 | 2.39 | 5.80 | 3.98 | 655.40 |
| 5.91 | 2.78 | 4.61 | 3.90 | 347.85 |
| 3.75 | 2.45 | 3.44 | 4.04 | 168.78 |
| 4.14 | 3.24 | 6.01 | 4.00 | 204.42 |
| 6.81 | 3.41 | 2.19 | 3.92 | 423.27 |
| 5.26 | 2.88 | 3.34 | 4.14 | 262.00 |
| 1.03 | 2.71 | 7.57 | 4.01 | 139.51 |
| 8.84 | 2.01 | 4.07 | 4.00 | 811.16 |
| 3.20 | 2.21 | 2.79 | 3.67 | 127.84 |
| 8.62 | 2.39 | 3.45 | 4.51 | 760.18 |
| 4.69 | 3.32 | 5.68 | 3.99 | 227.61 |
| 1.61 | 4.14 | 4.67 | 3.84 | 98.04 |
| 9.58 | 2.61 | 2.89 | 3.50 | 977.06 |
| 9.98 | 2.42 | 2.72 | 4.24 | 1109.20 |
| 1.87 | 1.86 | 3.57 | 3.96 | 101.99 |

Training Sample 2

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 8.13 | 2.95 | 5.61 | 3.87 | 676.06 |
| 1.70 | 2.86 | 4.19 | 4.13 | 91.37 |
| 5.53 | 2.89 | 5.05 | 3.78 | 288.47 |
| 7.85 | 3.27 | 5.56 | 4.08 | 604.44 |
| 9.24 | 2.36 | 2.25 | 3.93 | 899.95 |
| 9.80 | 2.22 | 2.79 | 4.27 | 1070.64 |
| 1.03 | 2.95 | 6.41 | 3.96 | 122.22 |
| 2.62 | 2.80 | 4.72 | 4.13 | 113.92 |
| 3.91 | 3.48 | 5.76 | 3.56 | 172.30 |
| 2.16 | 3.51 | 5.71 | 3.84 | 135.93 |
| 5.84 | 2.76 | 6.85 | 3.93 | 341.32 |
| 1.54 | 2.50 | 5.73 | 4.00 | 135.15 |
| 3.47 | 2.21 | 7.44 | 4.13 | 175.75 |
| 1.05 | 3.42 | 7.46 | 4.23 | 138.90 |
| 5.93 | 4.10 | 4.77 | 3.75 | 326.03 |
| 4.58 | 2.70 | 4.58 | 4.02 | 209.27 |
| 9.94 | 2.85 | 3.83 | 4.03 | 1107.42 |
| 5.62 | 2.13 | 2.21 | 3.95 | 269.32 |
| 6.23 | 3.15 | 6.72 | 3.41 | 369.31 |
| 7.53 | 3.02 | 6.41 | 4.18 | 569.89 |
| 2.09 | 3.02 | 4.92 | 4.11 | 107.54 |
| 8.32 | 2.25 | 3.83 | 3.69 | 669.13 |
| 1.87 | 1.86 | 3.57 | 3.96 | 101.99 |
| 8.72 | 3.36 | 7.87 | 3.99 | 836.10 |
| 4.05 | 2.40 | 6.34 | 3.84 | 192.66 |

Testing Sample

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| 9.98 | 1.62 | 4.55 | 3.72 | 1088.51 |
| 1.36 | 2.85 | 7.31 | 3.92 | 141.27 |
| 6.54 | 2.76 | 6.43 | 4.24 | 433.44 |
| 7.23 | 2.69 | 4.25 | 3.98 | 515.15 |
| 8.66 | 2.03 | 3.39 | 4.01 | 750.85 |
| 2.09 | 3.12 | 3.66 | 4.19 | 121.92 |
| 7.52 | 2.27 | 4.23 | 3.86 | 535.36 |
| 4.96 | 3.10 | 4.02 | 4.09 | 230.74 |
| 2.47 | 1.95 | 3.92 | 3.75 | 116.08 |
| 4.56 | 3.58 | 3.13 | 3.58 | 196.64 |
| 1.80 | 3.60 | 7.93 | 4.06 | 161.77 |
| 6.69 | 2.10 | 4.66 | 3.76 | 429.47 |
| 7.46 | 3.17 | 3.10 | 3.98 | 527.84 |
| 2.51 | 3.25 | 3.76 | 4.01 | 140.73 |
| 3.01 | 3.02 | 7.88 | 4.01 | 181.21 |
| 2.01 | 2.30 | 3.95 | 4.09 | 106.91 |
| 1.38 | 2.94 | 2.64 | 4.24 | 107.69 |
| 1.63 | 1.81 | 7.10 | 3.99 | 150.85 |
| 4.53 | 2.69 | 2.73 | 4.17 | 189.20 |
| 2.67 | 3.16 | 5.41 | 4.32 | 157.98 |
| 1.10 | 3.11 | 2.59 | 4.17 | 103.75 |
| 2.58 | 2.40 | 3.15 | 3.77 | 86.91 |
| 8.77 | 2.32 | 6.92 | 3.76 | 812.19 |
| 2.09 | 3.02 | 4.92 | 4.11 | 107.54 |
| 7.79 | 2.13 | 5.45 | 3.80 | 596.56 |

Model a

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 1088.51 | 702.50 | 149009.31 | 103.46 |
| 141.27 | 73.34 | 4614.39 | |
| 433.44 | 434.46 | 1.04 | |
| 515.15 | 487.11 | 786.37 | |
| 750.85 | 597.60 | 23487.06 | |
| 121.92 | 119.10 | 7.92 | |
| 535.36 | 509.59 | 664.09 | |
| 230.74 | 317.45 | 7518.87 | |
| 116.08 | 143.89 | 773.34 | |
| 196.64 | 288.38 | 8416.71 | |
| 161.77 | 100.47 | 3757.31 | |
| 429.47 | 446.13 | 277.75 | |
| 527.84 | 504.46 | 546.37 | |
| 140.73 | 146.90 | 38.06 | |
| 181.21 | 180.39 | 0.68 | |
| 106.91 | 113.71 | 46.35 | |
| 107.69 | 74.43 | 1106.14 | |
| 150.85 | 90.04 | 3698.04 | |
| 189.20 | 286.61 | 9488.30 | |
| 157.98 | 157.30 | 0.46 | |
| 103.75 | 57.34 | 2153.89 | |
| 86.91 | 151.09 | 4119.99 | |
| 812.19 | 606.05 | 42490.90 | |
| 107.54 | 119.32 | 138.77 | |
| 596.56 | 529.89 | 4444.31 | |

Model b

| Desired | Actual | SE | RMSE |
|---|---|---|---|
| 1088.51 | 692.59 | 156758.45 | 107.16 |
| 141.27 | 89.47 | 2683.08 | |
| 433.44 | 448.18 | 217.26 | |
| 515.15 | 497.10 | 325.71 | |
| 750.85 | 598.22 | 23297.08 | |
| 121.92 | 138.81 | 285.24 | |
| 535.36 | 517.84 | 306.98 | |
| 230.74 | 337.31 | 11356.96 | |
| 116.08 | 164.73 | 2366.91 | |
| 196.64 | 309.21 | 12671.74 | |
| 161.77 | 118.98 | 1830.34 | |
| 429.47 | 459.07 | 876.59 | |
| 527.84 | 513.12 | 216.68 | |
| 140.73 | 167.85 | 735.48 | |
| 181.21 | 202.16 | 438.99 | |
| 106.91 | 133.11 | 686.45 | |
| 107.69 | 90.68 | 289.48 | |
| 150.85 | 107.74 | 1858.63 | |
| 189.20 | 307.48 | 13991.31 | |
| 157.98 | 178.58 | 424.36 | |
| 103.75 | 71.60 | 1034.00 | |
| 86.91 | 172.19 | 7272.31 | |
| 812.19 | 605.88 | 42562.35 | |
| 107.54 | 139.03 | 991.98 | |
| 596.56 | 536.49 | 3608.14 | |

Average
RMSE: 105.31

# VITA

Michael Francis Cochrane is employed by the United States Army as a transportation engineer assigned to the Military Traffic Management Command Transportation Engineering Agency (MTMCTEA) in Newport News, Virginia. His current duties involve the development of techniques, procedures and technologies allowing the intermodal transportation system to support the deployment of military unit equipment during wartime.

Dr. Cochrane graduated from the United States Military Academy at West Point, NY in 1979, and served as a military intelligence officer from 1979 until 1984.

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